Elastic stiffness of fractured rock with multiple fracture-filling materials

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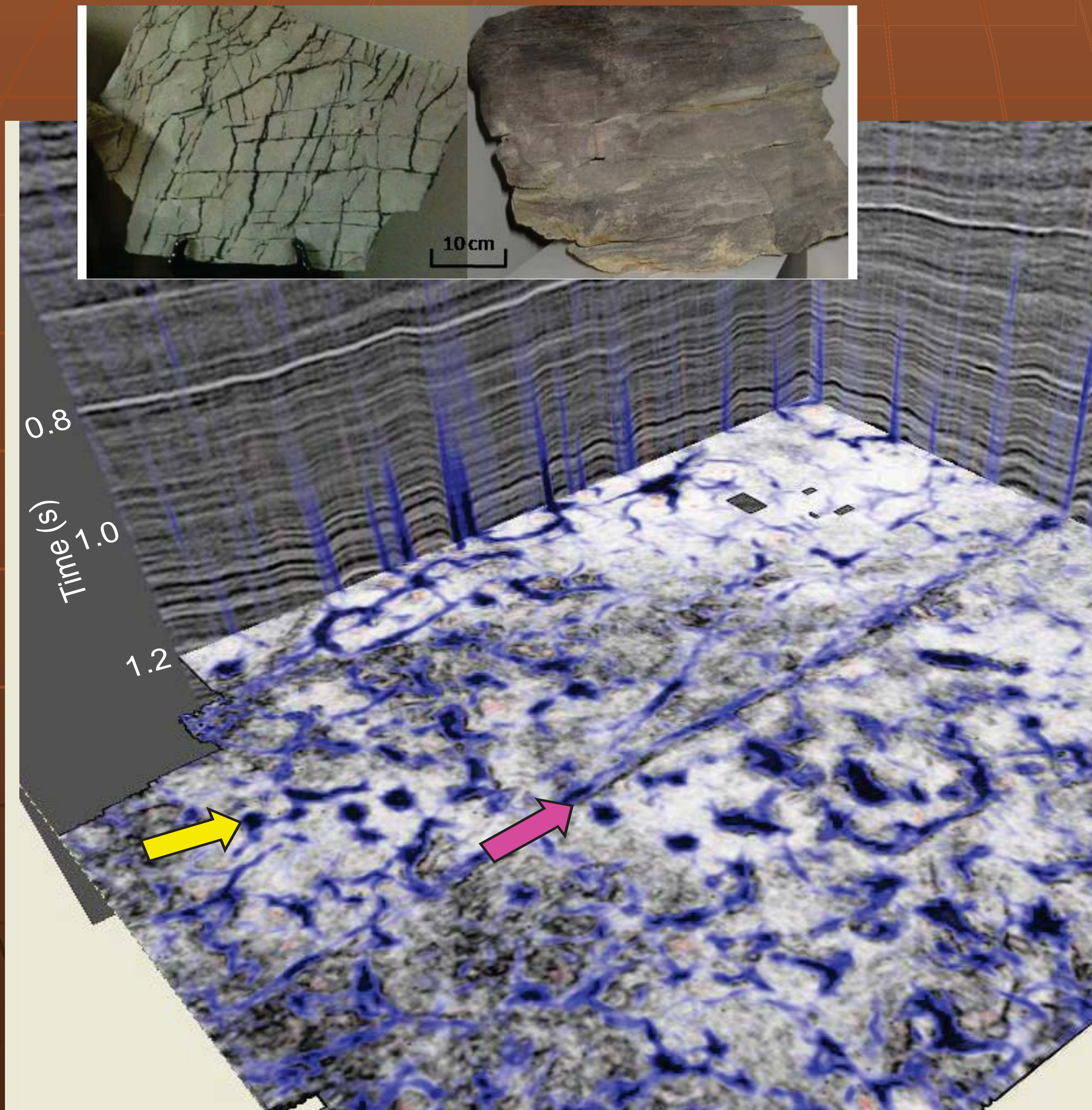
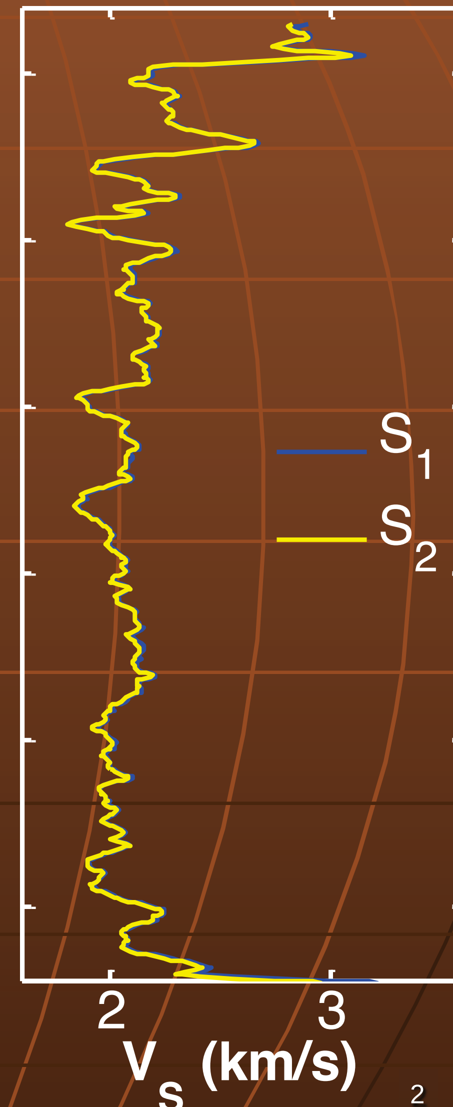
Motivation





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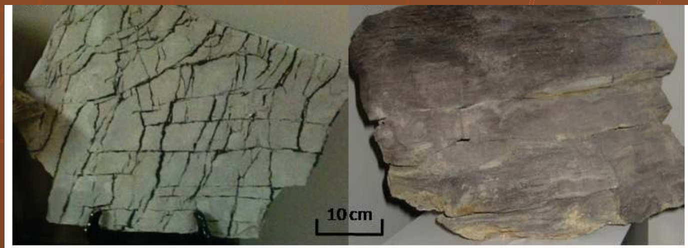

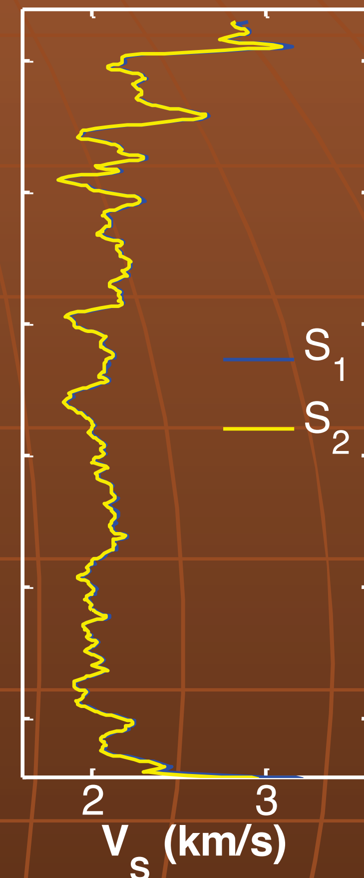

Is fracturing scale dependent?
Do localized well-log measurements miss them?

Are they fractures closed (just squeezed shut)?

Are the fractures fluid filled?

Are the fractures cemented (healed with a solid cement)?

Are some fractures cemented and others fluid filled?
Treat the cement as part of the pore space.

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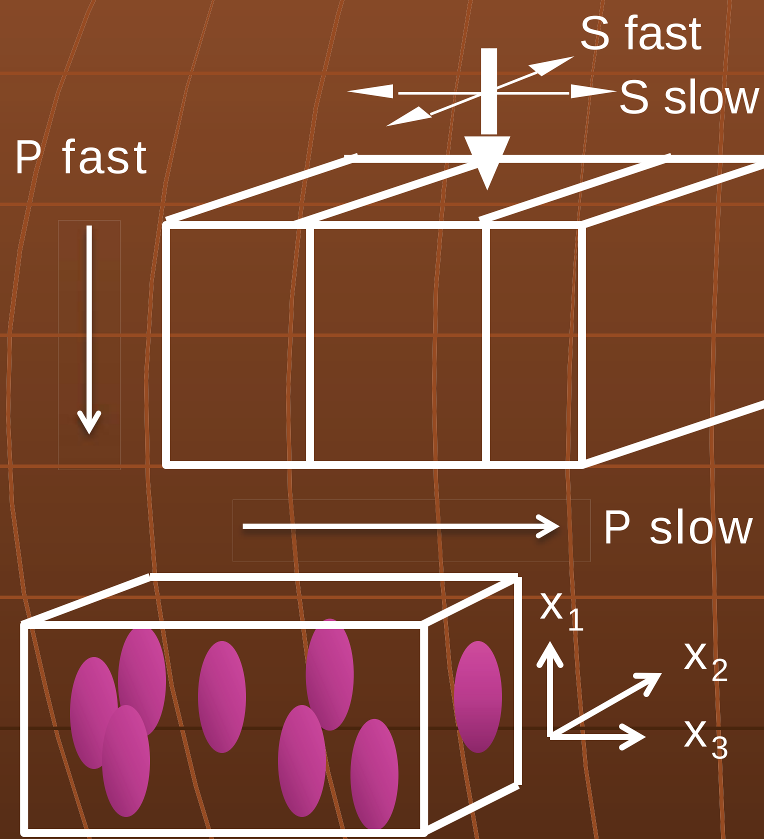
Effective medium for fractured rock



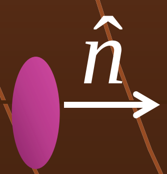
$$c_{ij}^{eff} = c_{ij}^{0} + c_{ij}^{1}$$

$$c_{33}^{1} = -\frac{(\lambda + 2\mu)^2}{\mu}\varepsilon_{cd}U_3$$

$$c_{11}^{1} = -\frac{\lambda^2}{\mu}\varepsilon_{cd}U_3$$

$$c_{13}^{1} = -\frac{\lambda(\lambda + 2\mu)}{\mu}\varepsilon_{cd}U_3$$

$$c_{44}^{1} = -\mu\varepsilon_{cd}U_1; \qquad c_{66}^{1} = 0$$



$$\varepsilon_{cd} = \frac{N}{V}a^3 = \frac{3\phi}{4\pi\alpha} = \text{crack density}$$

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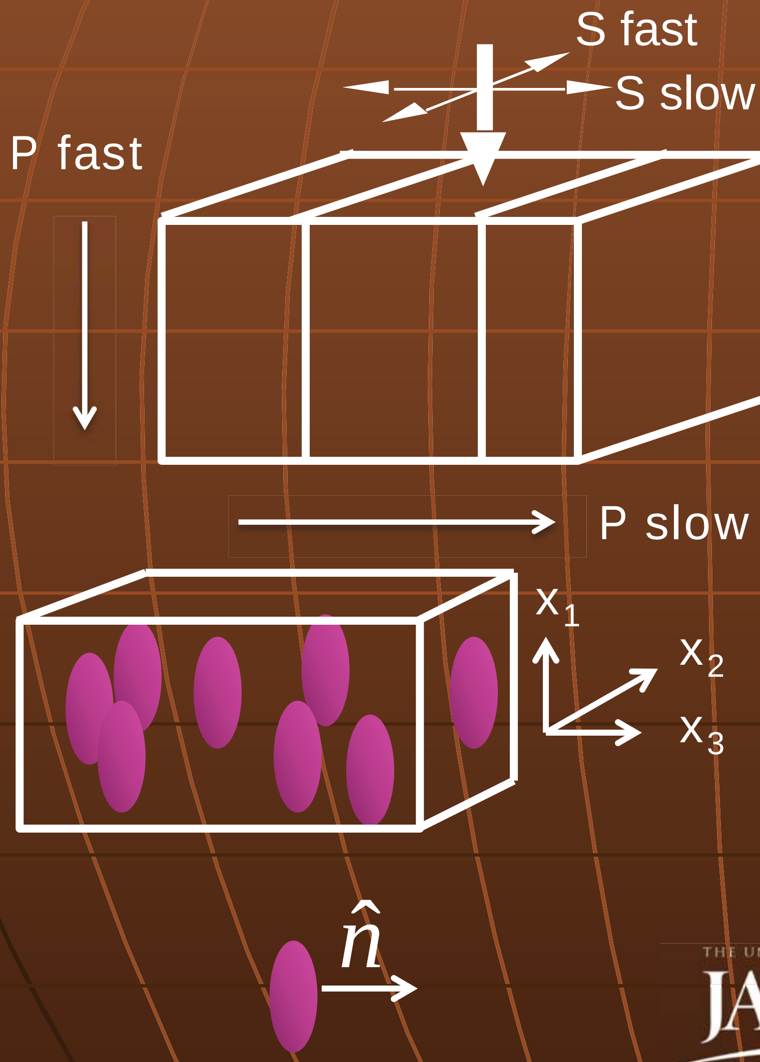

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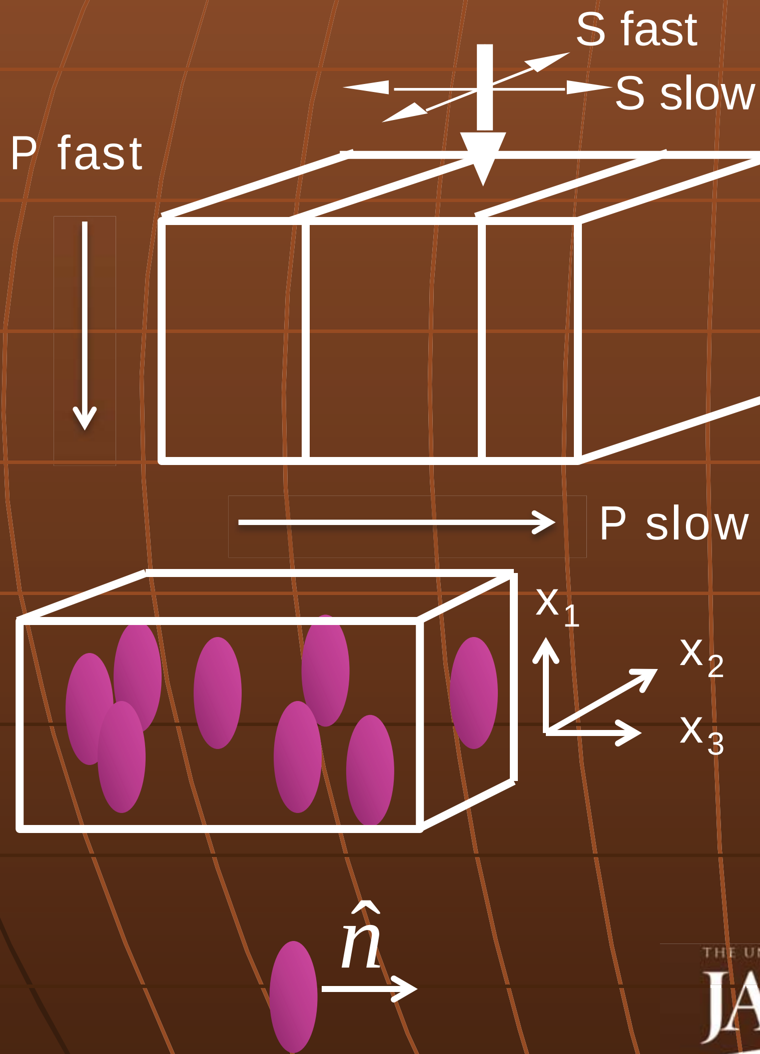

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$$U_1 = \frac{16}{3}\left(\frac{\lambda+2\mu}{3\lambda+4\mu}\right)\left(\frac{1}{1+M}\right)$$

$$U_3 = \frac{4}{3}\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\left(\frac{1}{1+\kappa}\right)$$

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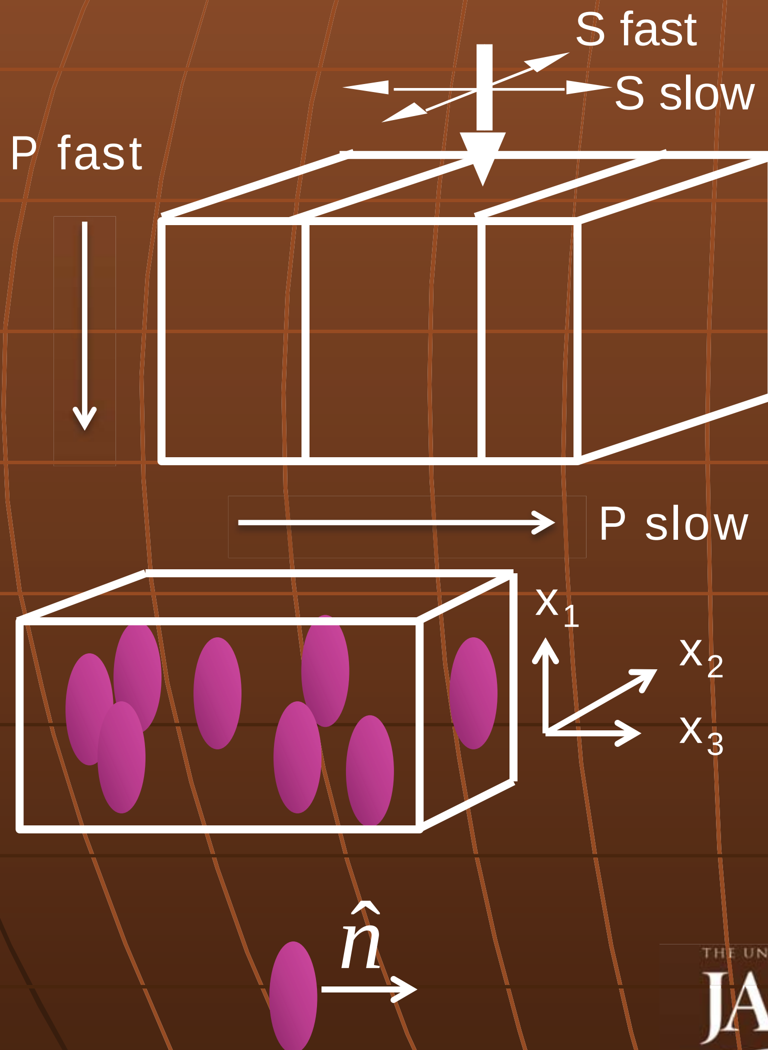

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$$U_3 = \frac{4}{3}\left(\frac{\lambda + 2\mu}{\lambda + \mu}\right)\left(\frac{1}{1+\kappa}\right)$$

$$M = \frac{4\mu'}{\pi\alpha\mu}\left(\frac{\lambda + 2\mu}{3\lambda + 4\mu}\right)$$

$$\kappa = \left(\frac{K' + \frac{4}{3}\mu'}{\pi\alpha\mu}\right)\left(\frac{\lambda + 2\mu}{\lambda + \mu}\right)$$

K', μ' bulk and shear moduli of inclusion material

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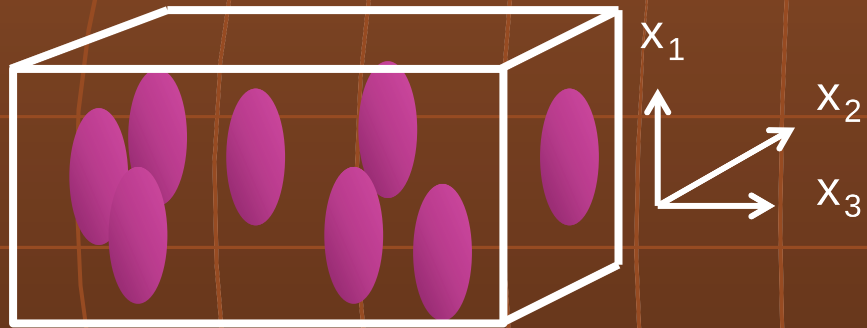
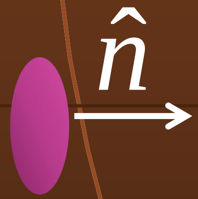
Effective fracture fill





Define

$$\phi = f_c\phi + (1 - f_c)\phi = \phi_c + \phi_f$$

$$\varepsilon_{cd} = \frac{3}{4\pi}\frac{\left(\phi_c + \phi_f\right)}{\alpha}$$

$$M = \left(\frac{\lambda + 2\mu}{3\lambda + 4\mu}\right)\left(\frac{4}{\pi\mu}\right)\left(\frac{\mu'}{\alpha}\right)$$

$$\kappa = \left(\frac{\lambda + 2\mu}{\lambda + \mu}\right)\left(\frac{1}{\pi\mu}\right)\left(\frac{K' + \frac{4}{3}\mu'}{\alpha}\right)$$

$$\left(\frac{\mu'}{\alpha}\right) = f_c\frac{\mu_c}{\alpha} + (1 - f_c)\frac{\mu_f}{\alpha} = f_c\frac{\mu_c}{\alpha}$$

$$\left(\frac{K' + \frac{4}{3}\mu'}{\alpha}\right) = f_c\left(\frac{K_c + \frac{4}{3}\mu_c}{\alpha}\right) + (1 - f_c)\frac{K_f}{\alpha}$$

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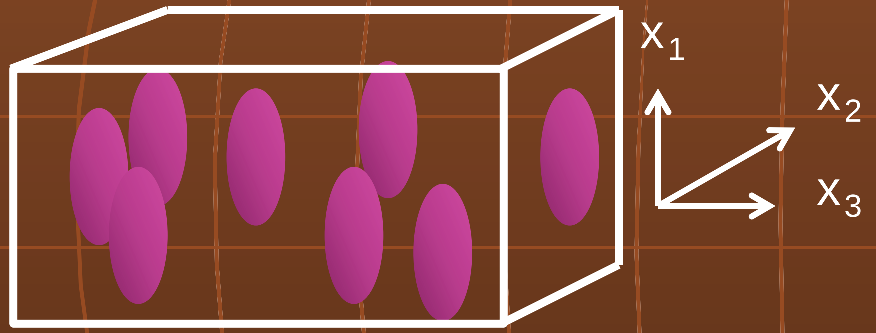
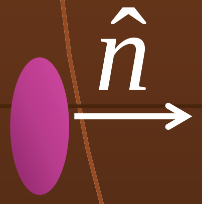
Effective fracture fill





Voigt Average of the cement and fluid

Define

$$\phi = f_c\phi + (1 - f_c)\phi = \phi_c + \phi_f$$

$$\varepsilon_{cd} = \frac{3}{4\pi}\frac{(\phi_c + \phi_f)}{\alpha}$$

$$M = \left(\frac{\lambda + 2\mu}{3\lambda + 4\mu}\right)\left(\frac{4}{\pi\mu}\right)\left(\frac{\mu'}{\alpha}\right)$$

$$\kappa = \left(\frac{\lambda + 2\mu}{\lambda + \mu}\right)\left(\frac{1}{\pi\mu}\right)\left(\frac{K' + \frac{4}{3}\mu'}{\alpha}\right)$$

$$\left(\frac{\mu'}{\alpha}\right) = f_c\frac{\mu_c}{\alpha} + (1 - f_c)\frac{\mu_f}{\alpha} = f_c\frac{\mu_c}{\alpha}$$

$$\left(\frac{K' + \frac{4}{3}\mu'}{\alpha}\right) = f_c\left(\frac{K_c + \frac{4}{3}\mu_c}{\alpha}\right) + (1 - f_c)\frac{K_f}{\alpha}$$

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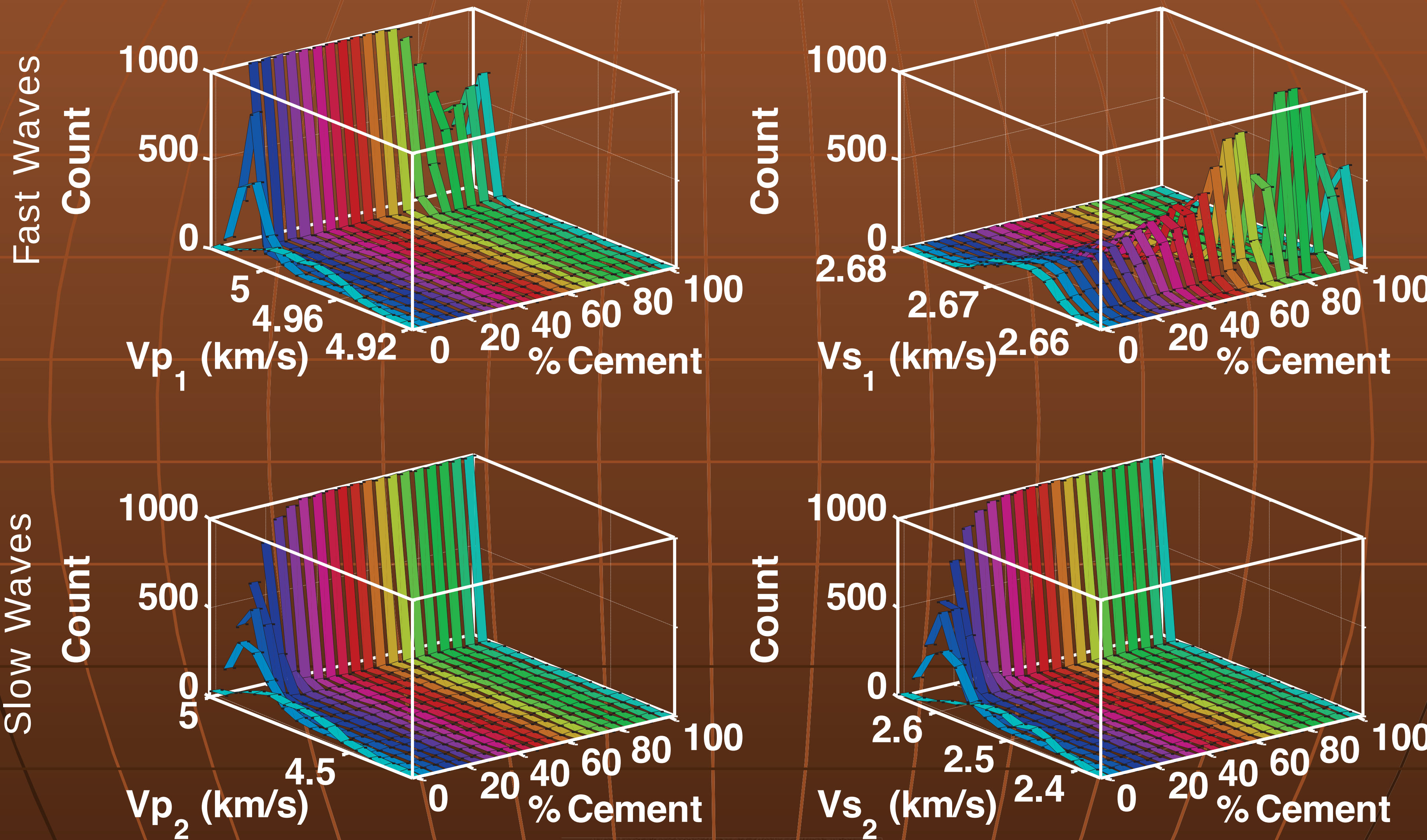
Effective fracture fill



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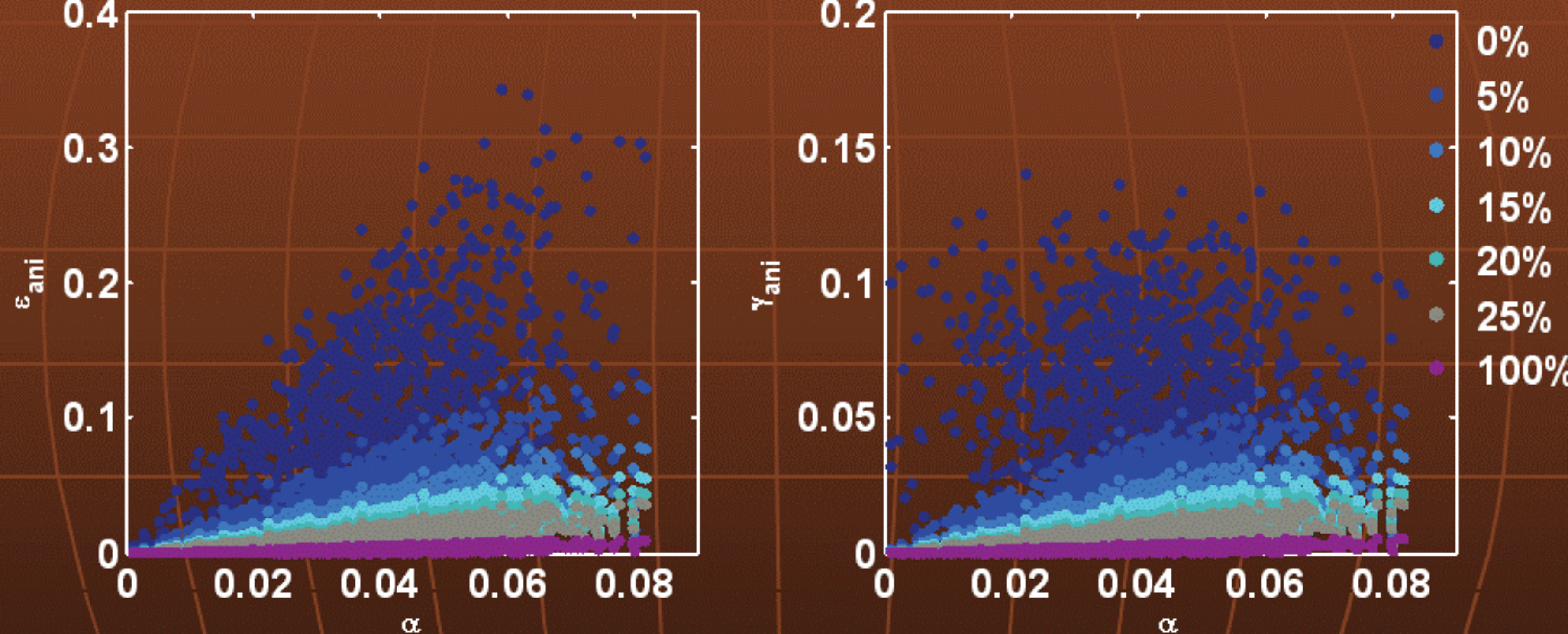

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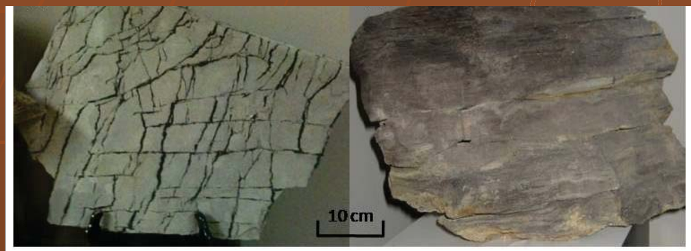
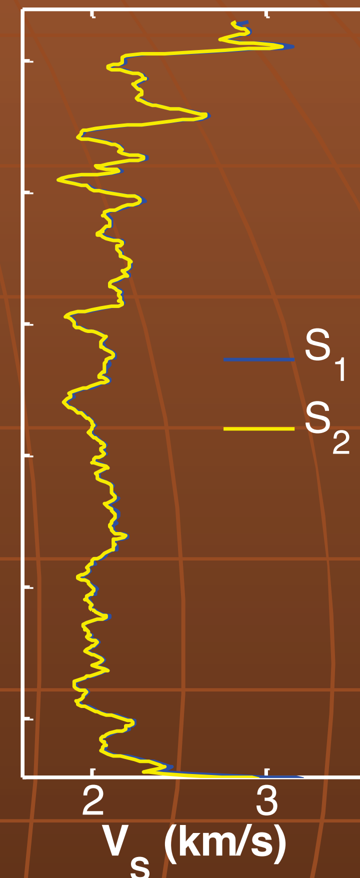
Motivation





Is fracturing scale dependent?
Do localized well-log measurements miss them?

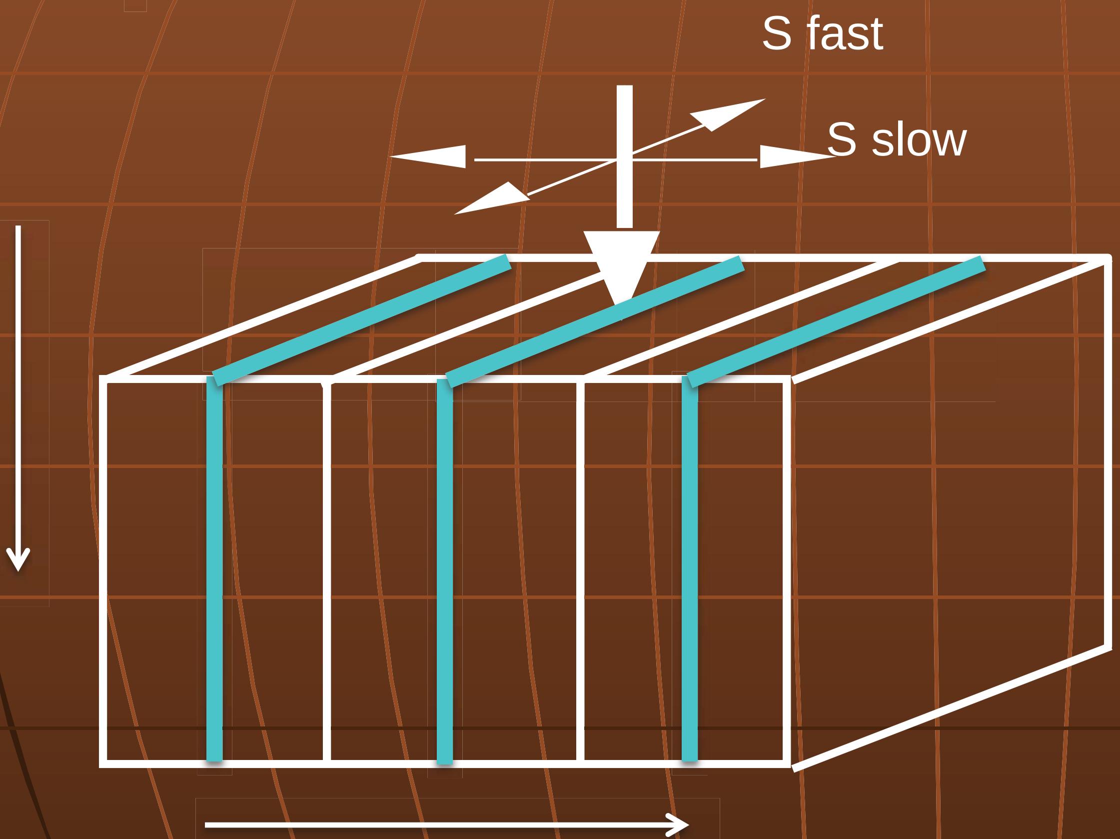
Are they fractures closed (just squeezed shut)?

Are the fractures fluid filled?

Are the fractures cemented (healed with a solid cement)?

Are some fractures cemented and others fluid filled?
Treatment system as 2 sets of parallel fractures.

Treat the background has having cement along with fluid-filled fractures.



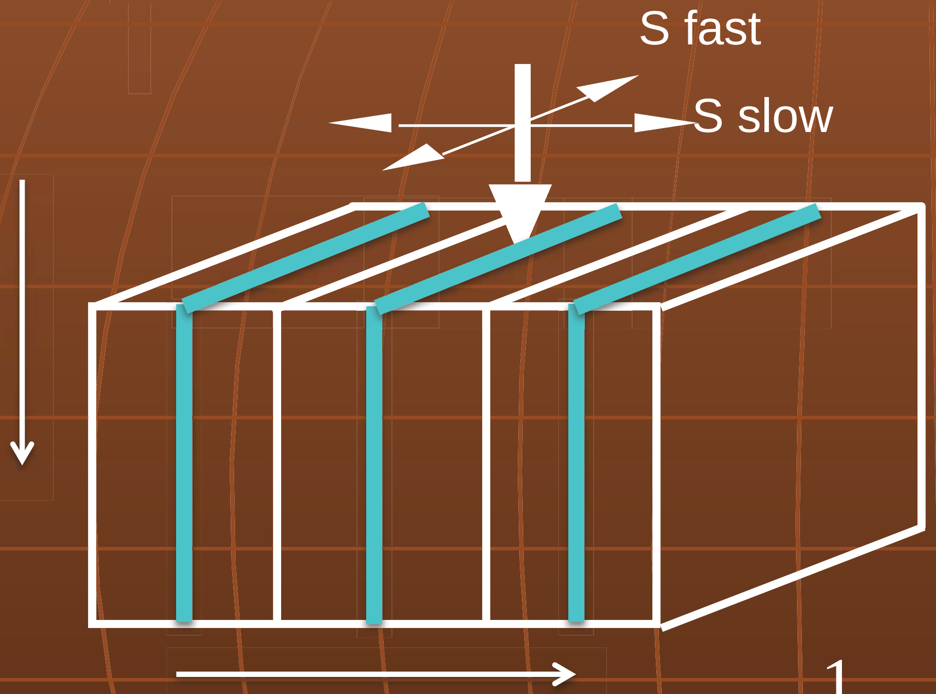
HTI background with (say fluid-filled fractures) with additional set of fractures (cement filled) aligned on same symmetry axis as the background.

Can this be solved?

Not with the Hudson model. assumes isotropic background.

Eshelby solution?

Treat the background has having cement along with fluid-filled fractures.



Compliance tensor for an inclusion embedded in a general anisotropic background.

Mura (1987) solved this analytically up to TI symmetry.

$$S_{ijkl} = \frac{1}{8\pi} L_{0_{mnkl}} \int_{-1}^{1} d\psi_3 \int_{0}^{2\pi} [G_{imjn}(\bar{\vartheta}) + G_{jmin}(\bar{\vartheta})] d\omega$$

L_0 is the stiffness tensor.

G is Green's function.

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Treat the background has having cement along with fluid-filled fractures.

$$S_{ijkl} = \frac{1}{8\pi} L_{0_{mnkl}} \int_{-1}^{1} d\psi_3 \int_{0}^{2\pi} [G_{imjn}(\bar{\vartheta}) + G_{jmin}(\bar{\vartheta})] d\omega$$

$$G_{imjn}(\bar{\vartheta}) = \bar{\vartheta}_k \bar{\vartheta}_l N_{ij}(\bar{\vartheta}) / D(\bar{\vartheta})$$

$$\bar{\vartheta}_i = \psi_i / a_i; \quad \psi_1 = (1-\psi_3^2)^{1/2} \cos\omega;$$

$$\psi_2 = (1-\psi_3^2)^{1/2} \sin\omega; \quad \psi_3 = \psi_3;$$

$$D(\bar{\vartheta}) = \varepsilon_{mnl} K_{m1} K_{n2} K_{l3};$$

$$N_{ij}(\bar{\vartheta}) \tfrac{1}{2} \varepsilon_{ikl} \varepsilon_{jmn} K_{km} K_{\ln};$$

$$K_{ik} = L_{0_{mnkl}} \bar{\vartheta}_j \bar{\vartheta}_l$$

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Treat the background has having cement along with fluid-filled fractures.

For anything above a TI background, numerical solutions must be included (Gavazzi and Lagoudas, 1990) through a Gaussian quadrature formulation. This is an area of future work. But, it is strictly valid for a single inclusion. It must be validated for multiple inclusions

$$S_{ijkl} = \frac{1}{8\pi} L_{0_{mnkl}} \int_{-1}^{1} d\psi_3 \int_{0}^{2\pi} [G_{imjn}(\bar{\vartheta}) + G_{jmin}(\bar{\vartheta})] d\omega$$

$$S_{ijkl} = \frac{1}{8\pi} \sum_{p=1}^{M} \sum_{q=1}^{N} L_{0_{mnkl}} [G_{imjn}(\omega_q, \psi_{3p}) + G_{jmin}(\omega_q, \psi_{3p})] W_{pq}$$

L_0 is the stiffness tensor. G is Green's function. W_{pq} are Gaussian weights.

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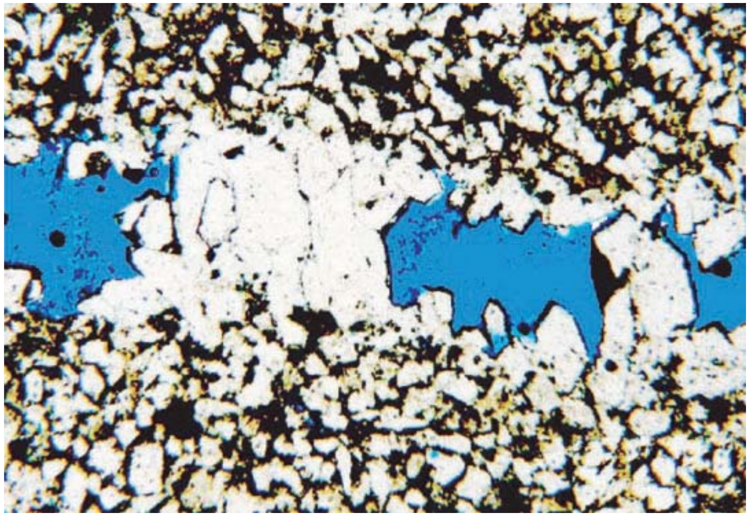
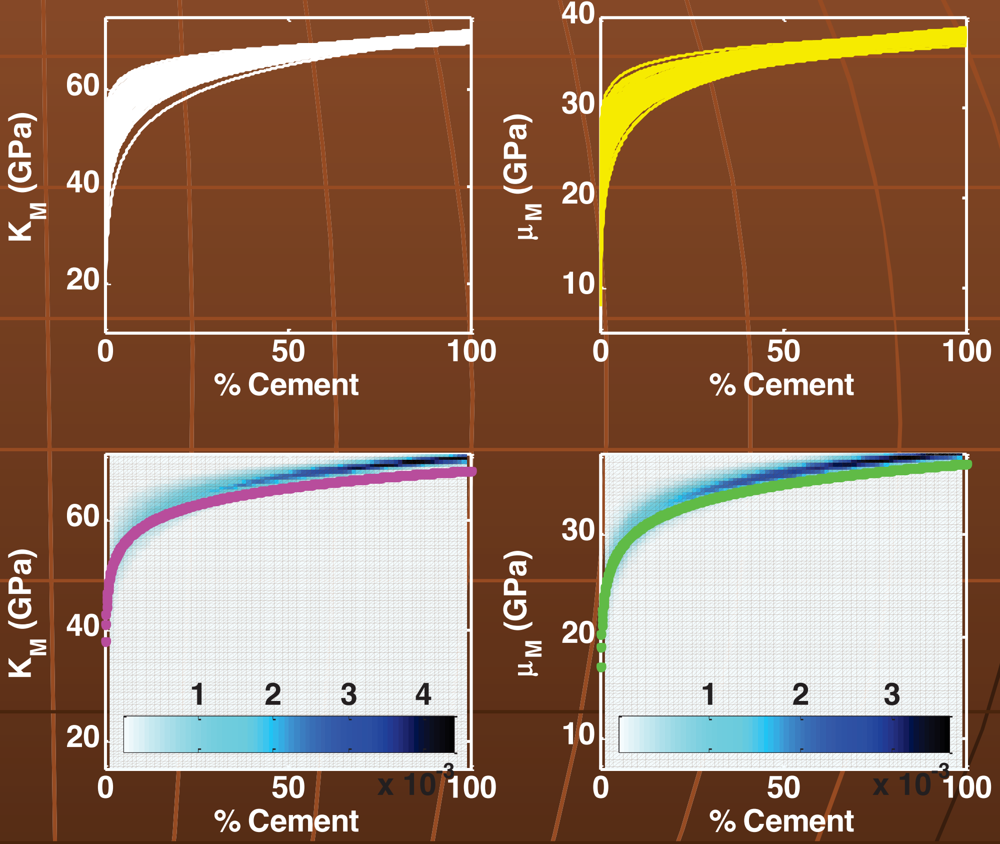
A stochastic approach

Simulate a series of background (isotropic) moduli that depend on cement content.



bridge in an open fracture, Cretaceous Travis Peak Formation, east Texas (from Laubach 2003).



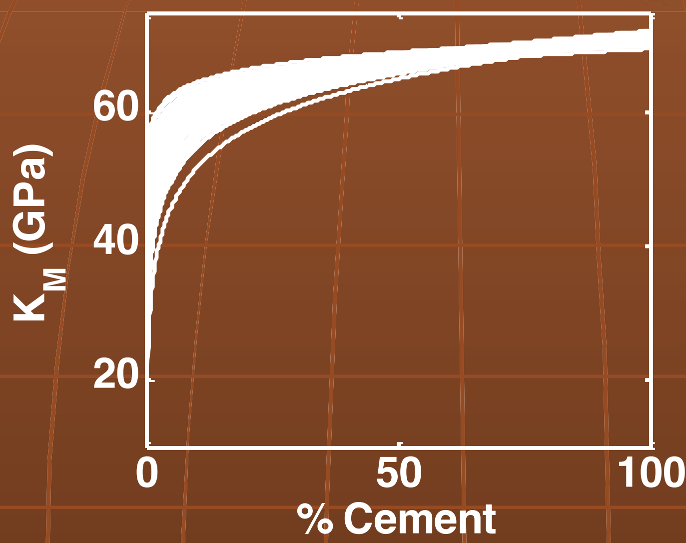
A stochastic approach

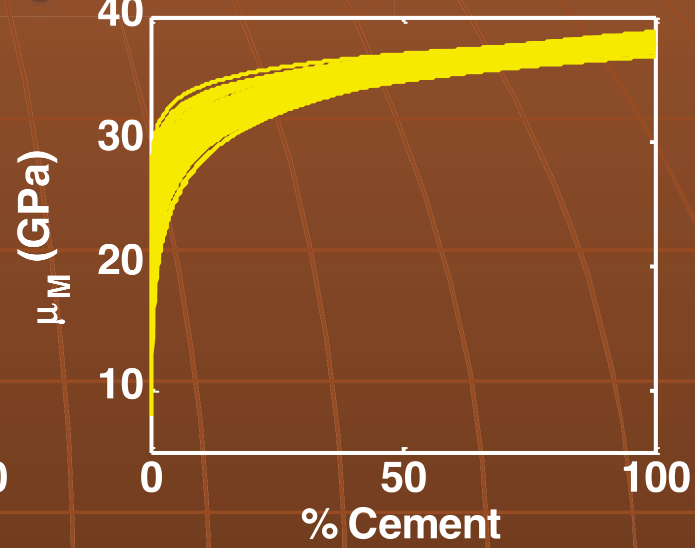
Simulate a series of background (isotropic) moduli that depend on cement content.

Also simulate correlated values of background density, fracture shape, and fracture density.

Advantage:
This is fast.

Disadvantage:
Not knowing the end points/means.





$$K_{\mathrm{M}} = \frac{\ln(f_c) + z_1}{z_2} + K^0_{\mathrm{cem}}$$

$$\mu_{\mathrm{M}} = \frac{\ln(f_c) + z_1}{z_3} + \mu^0_{\mathrm{cem}}$$

$$z_1 = \left|\min(\ln(f_c))\right|$$

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$$z_2 = \frac{\max(\ln(f_c)) + z_1}{K^1_{\mathrm{cem}} - K^0_{\mathrm{cem}}}$$

$$z_3 = \frac{\max(\ln(f_c)) + z_1}{\mu^1_{\mathrm{cem}} - \mu^0_{\mathrm{cem}}}$$

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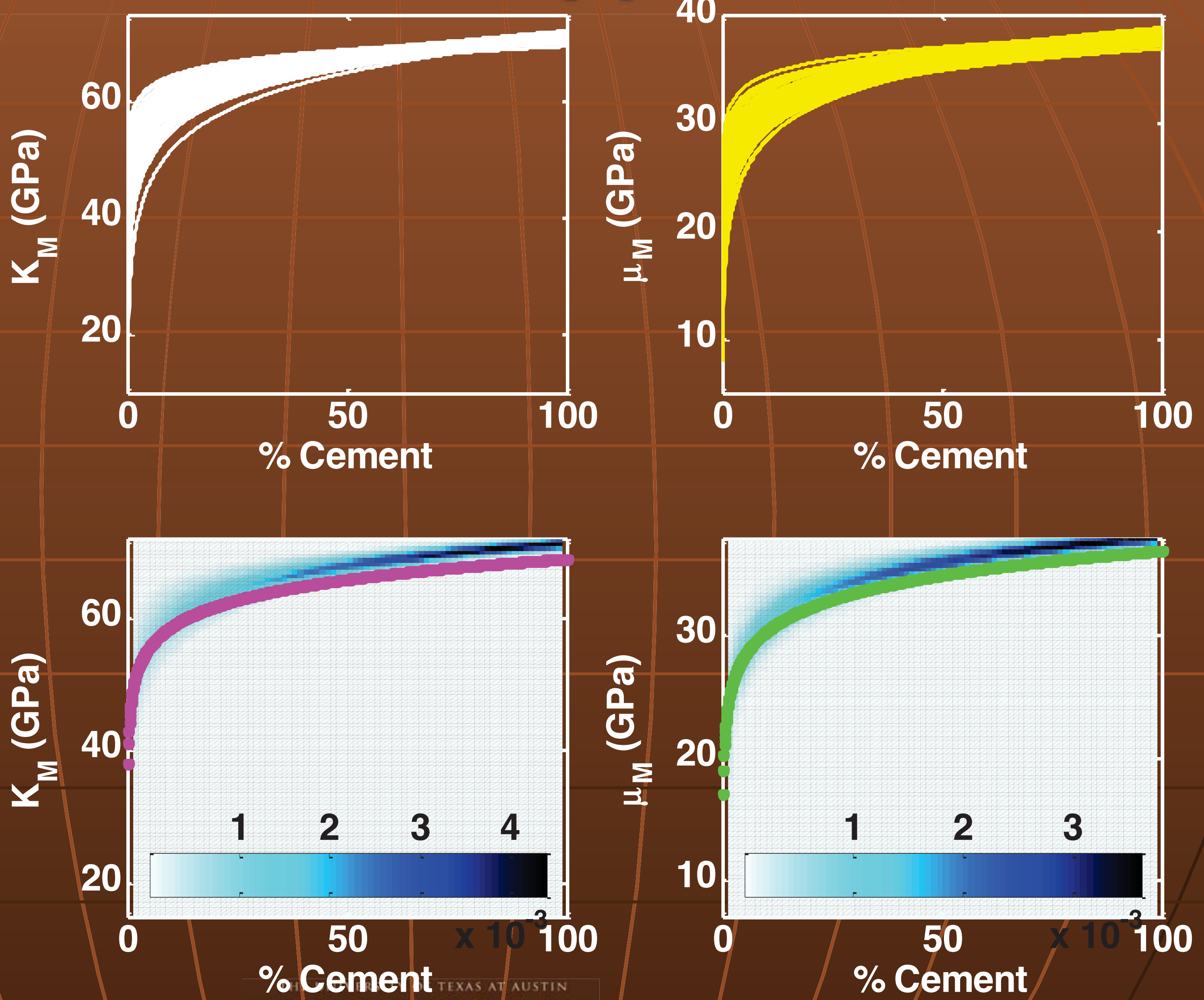

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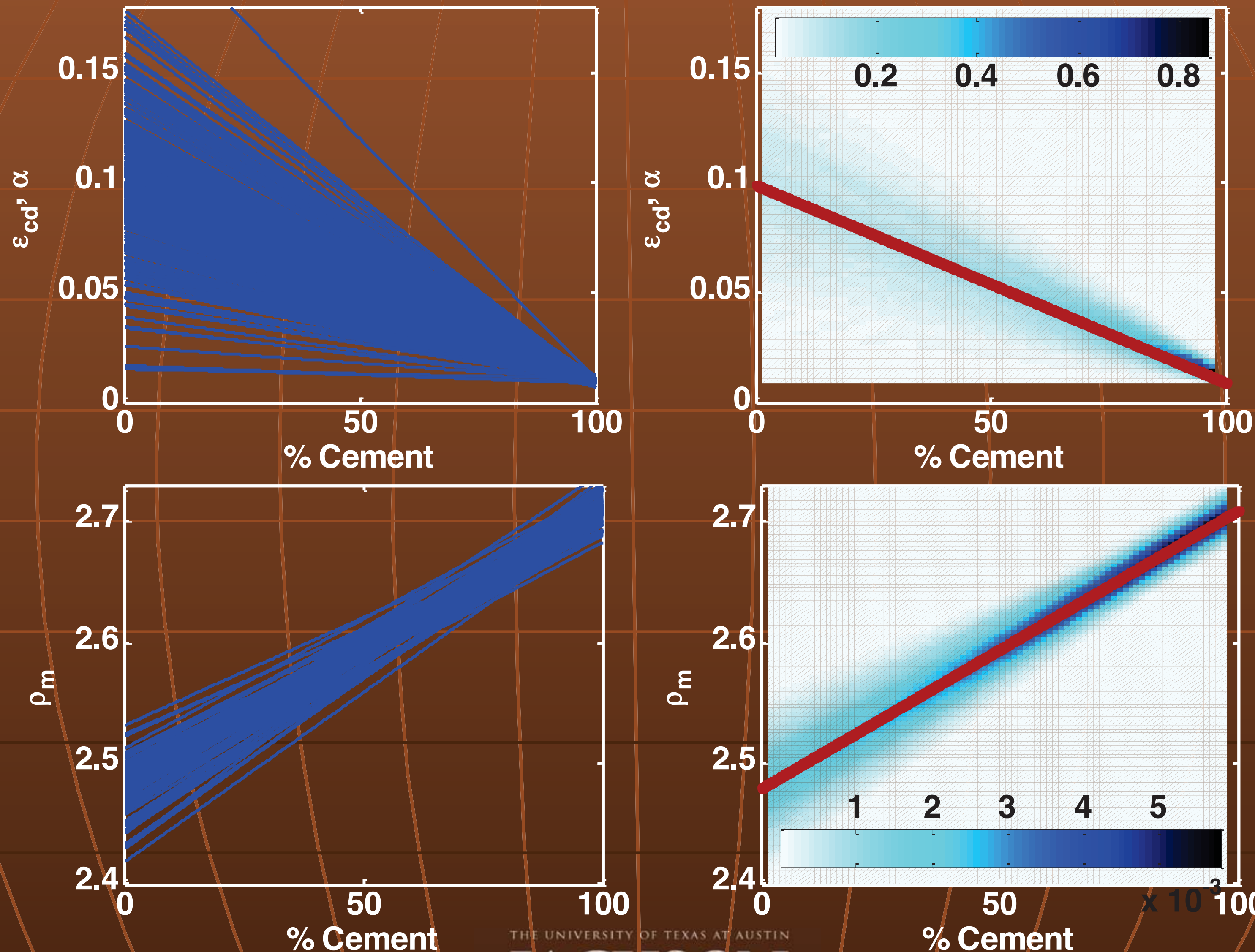
Correlated fracture parameters



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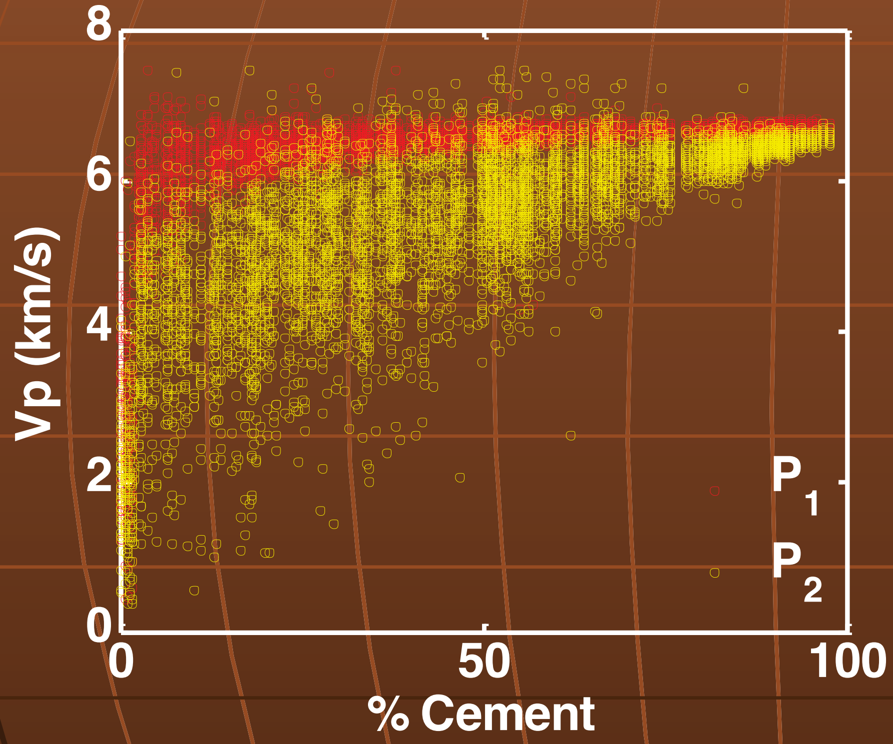
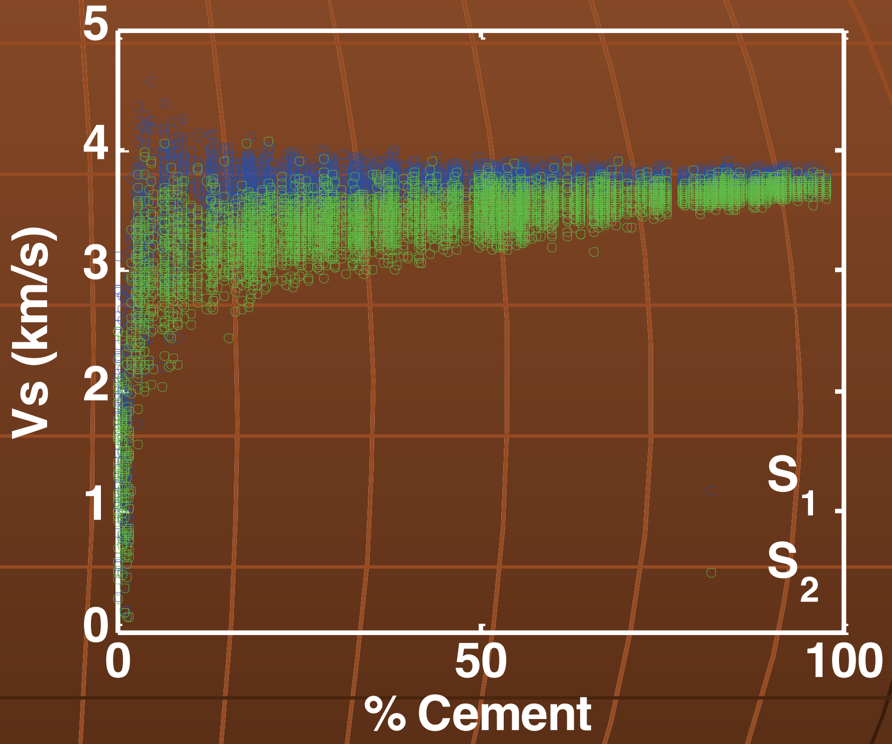
Fast and slow velocities





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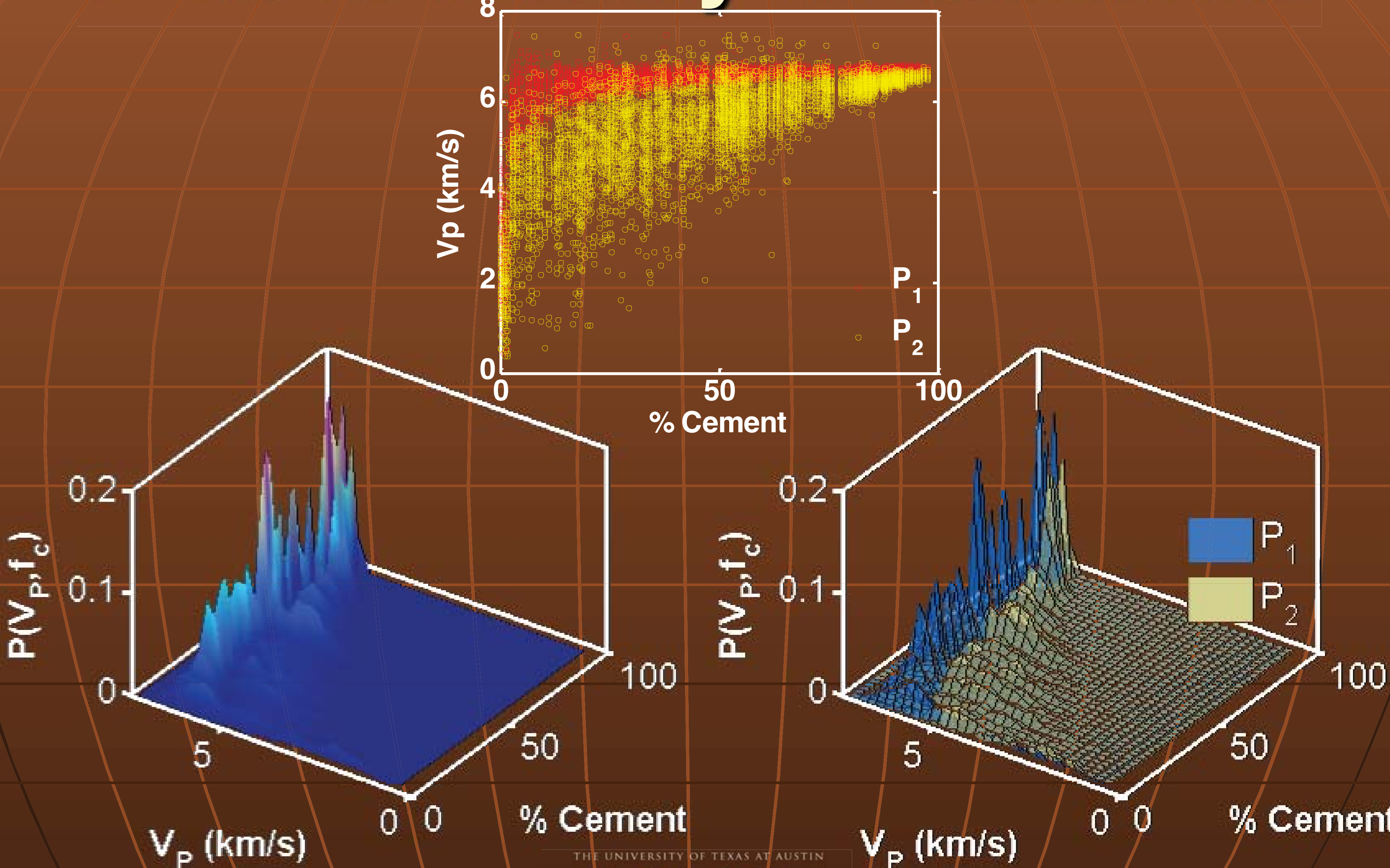
P-wave velocity distributions



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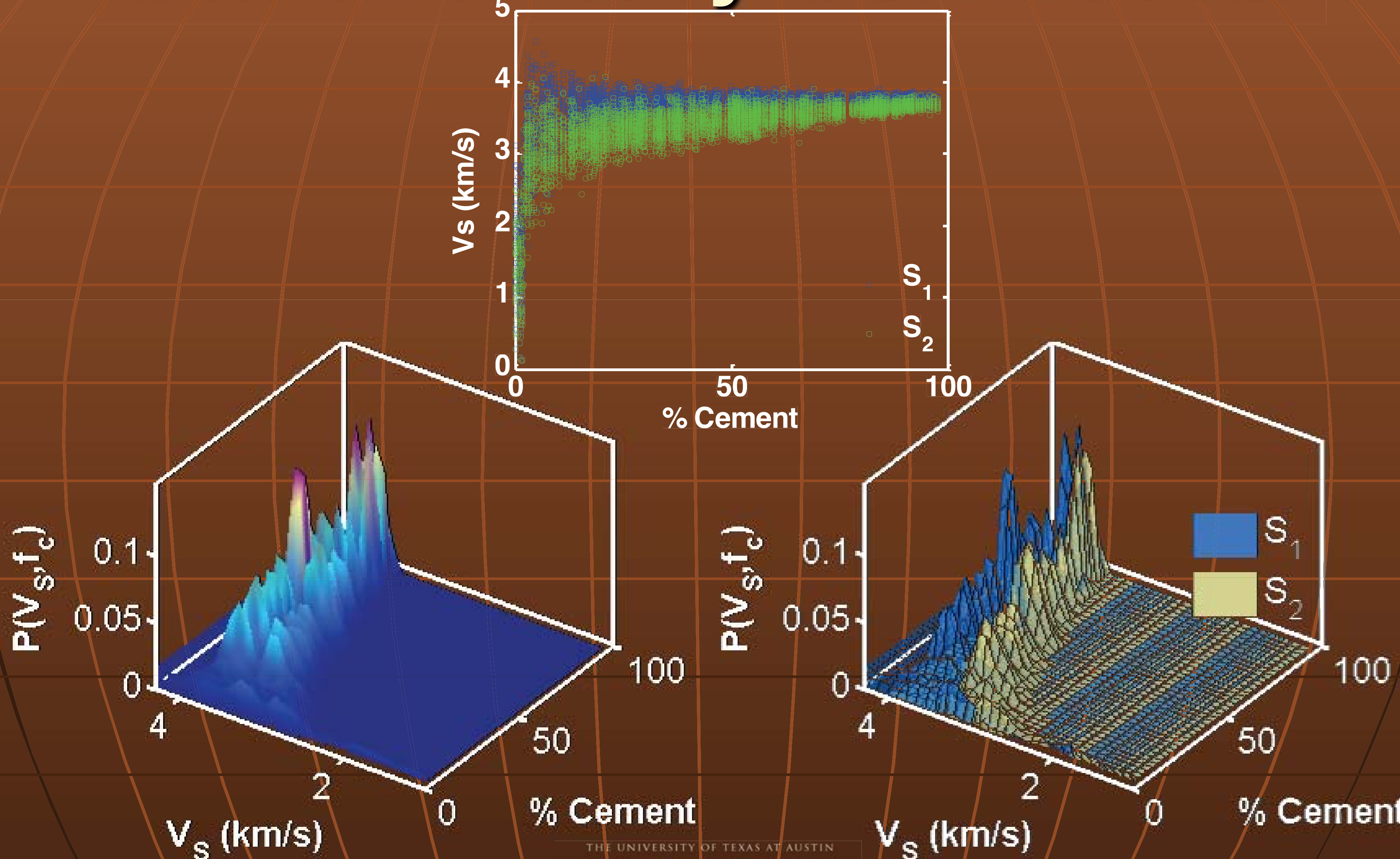
S-wave velocity distributions



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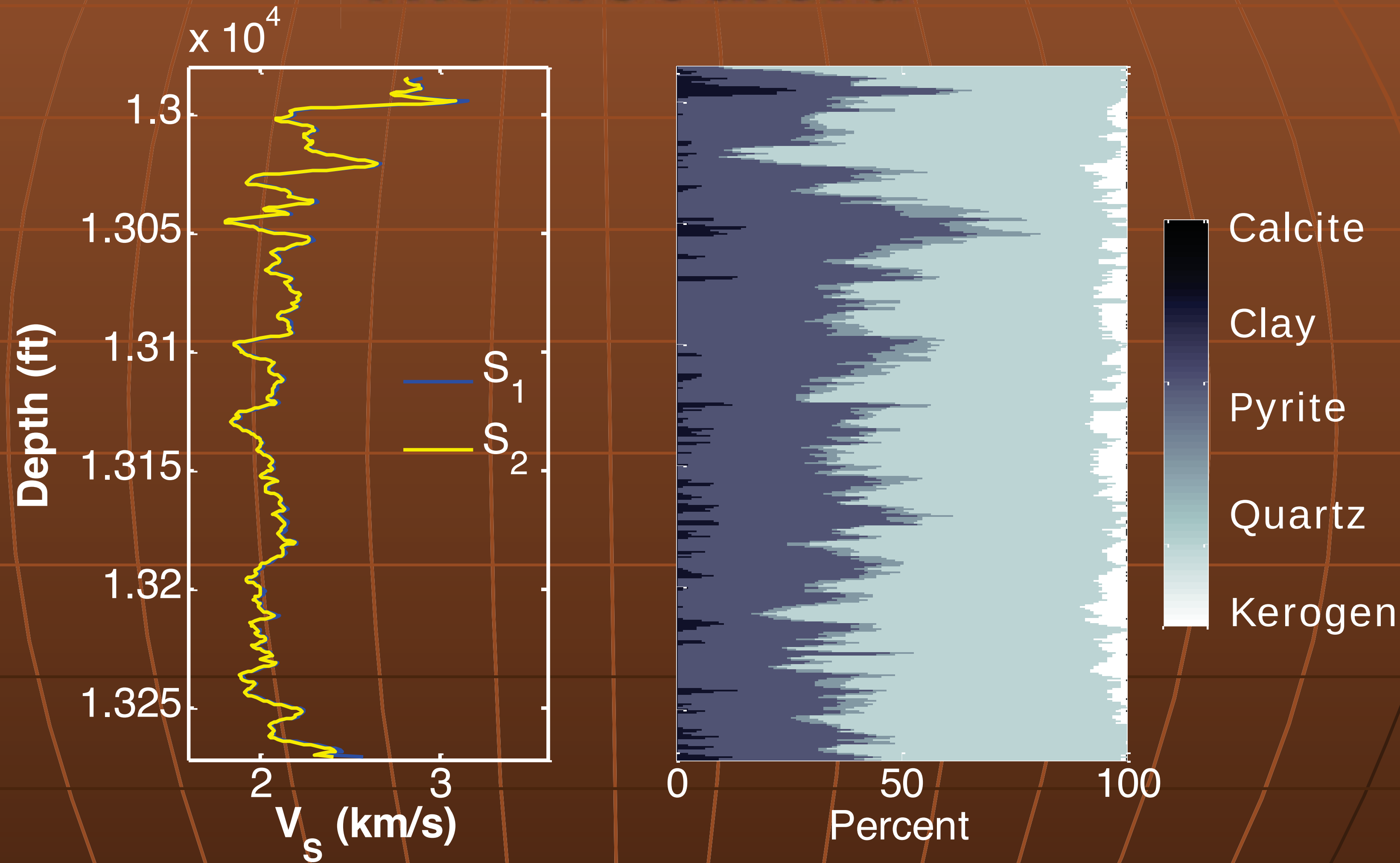
Real data comparison
The Woodford



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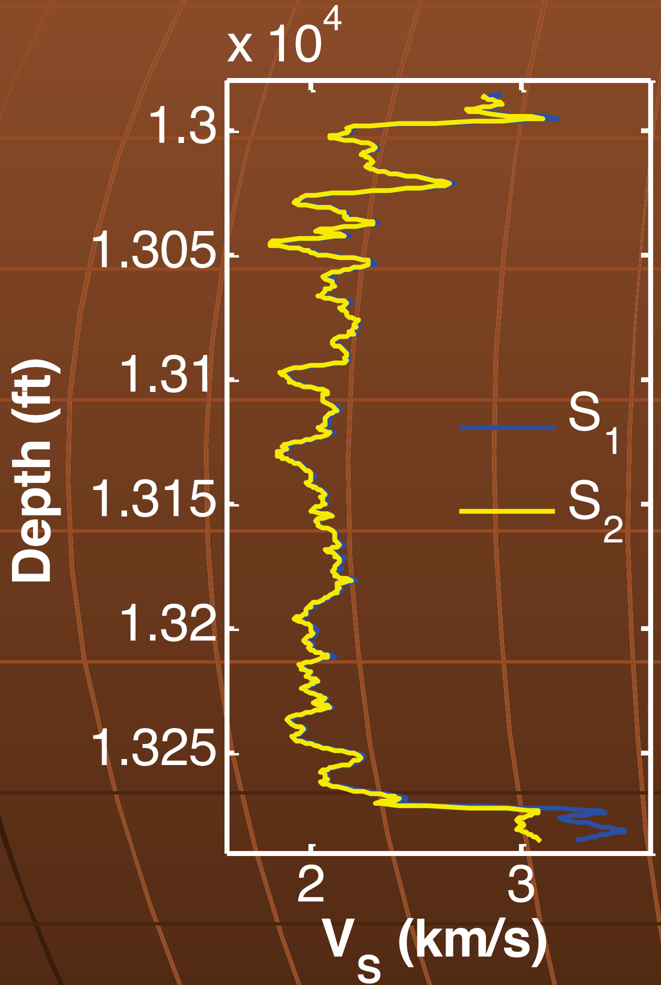

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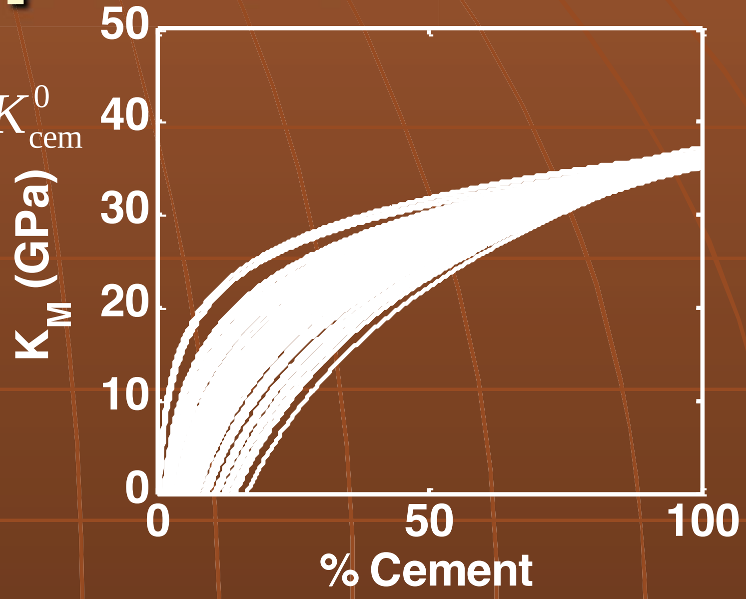
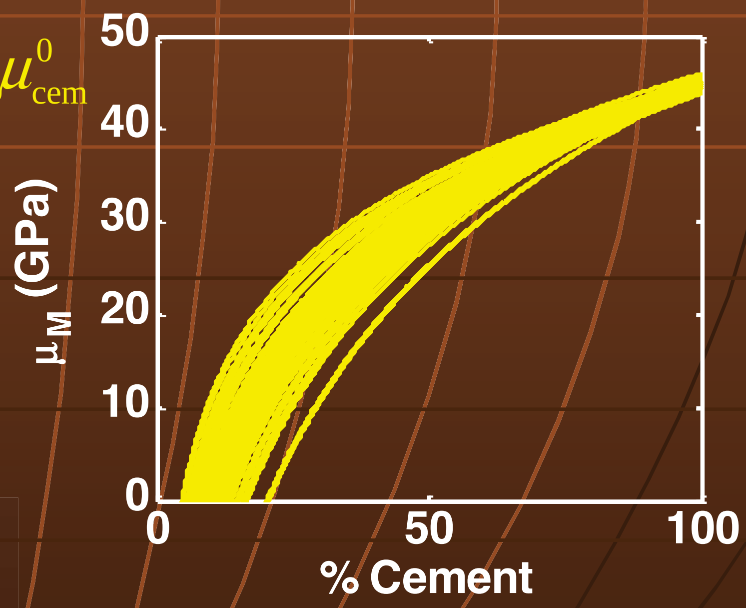


$$\mu_{\mathrm{M}} = \frac{\ln(f_c) + z_1}{z_3} + \mu_{\mathrm{cem}}^0$$



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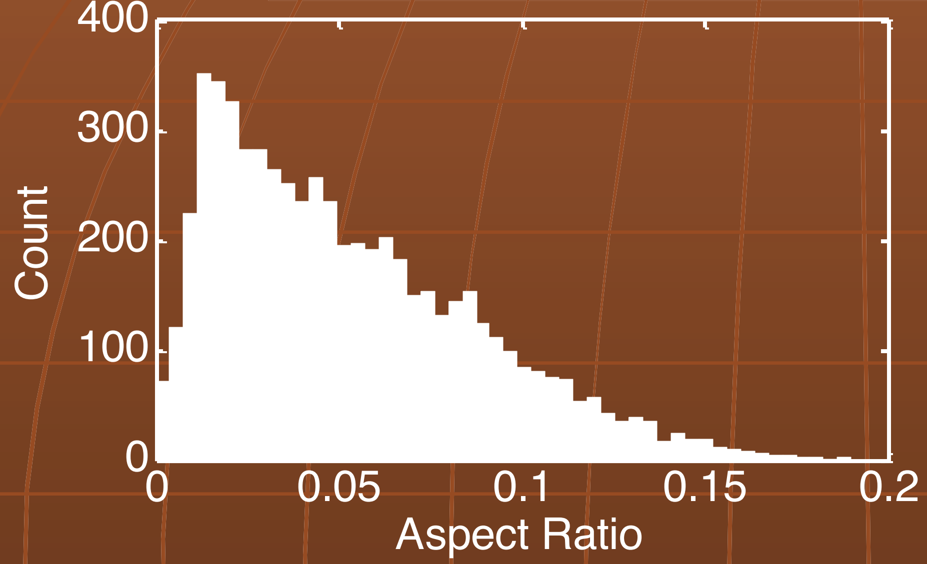

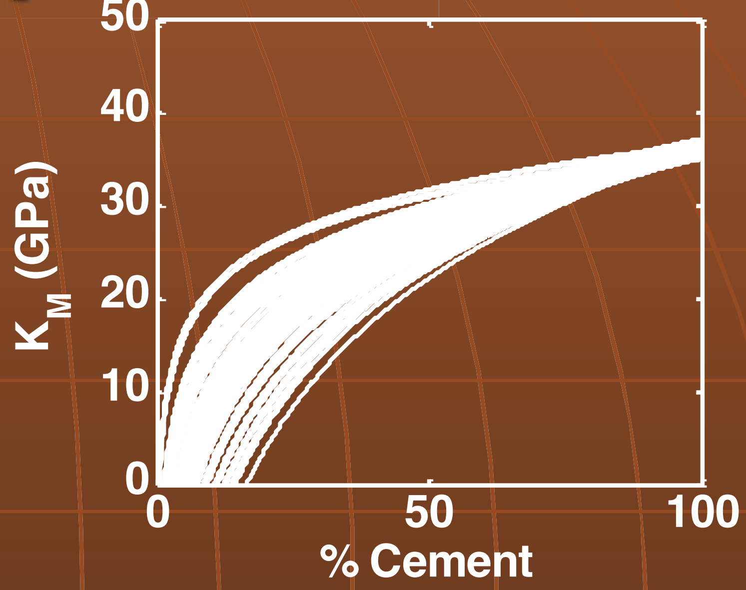

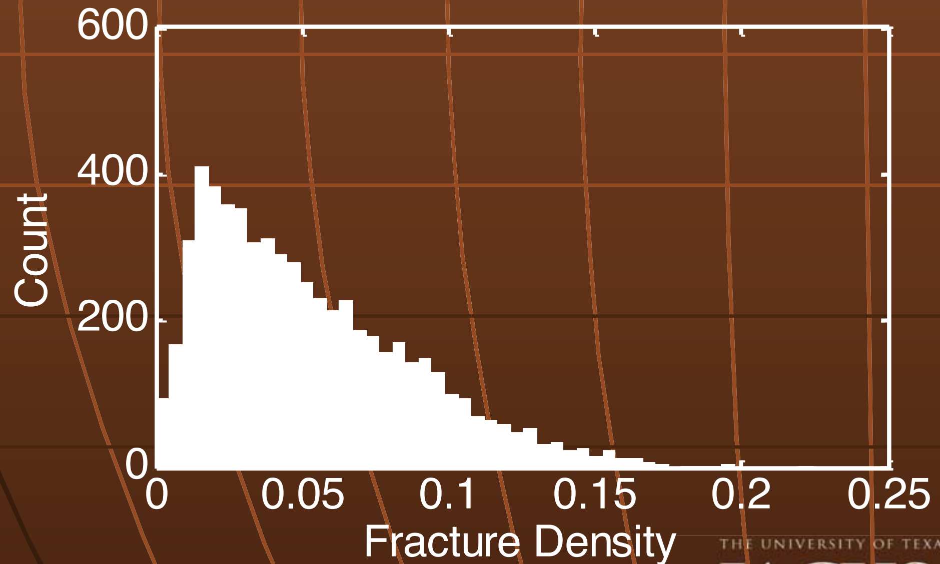

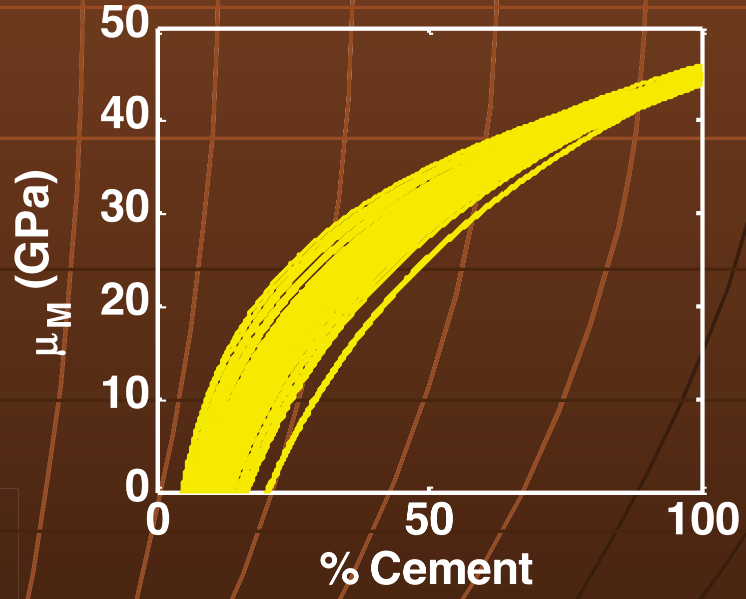

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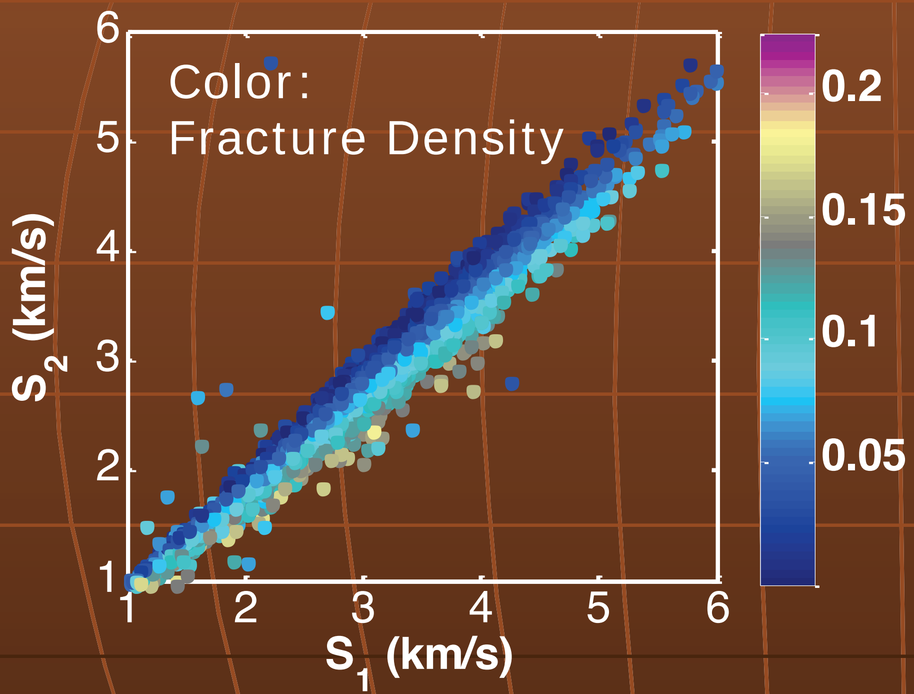

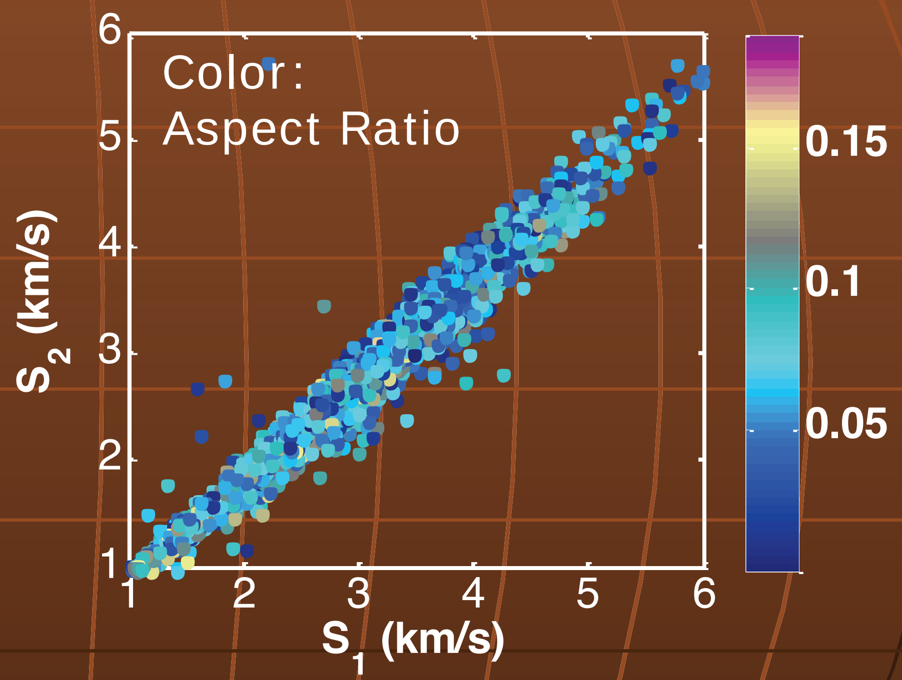

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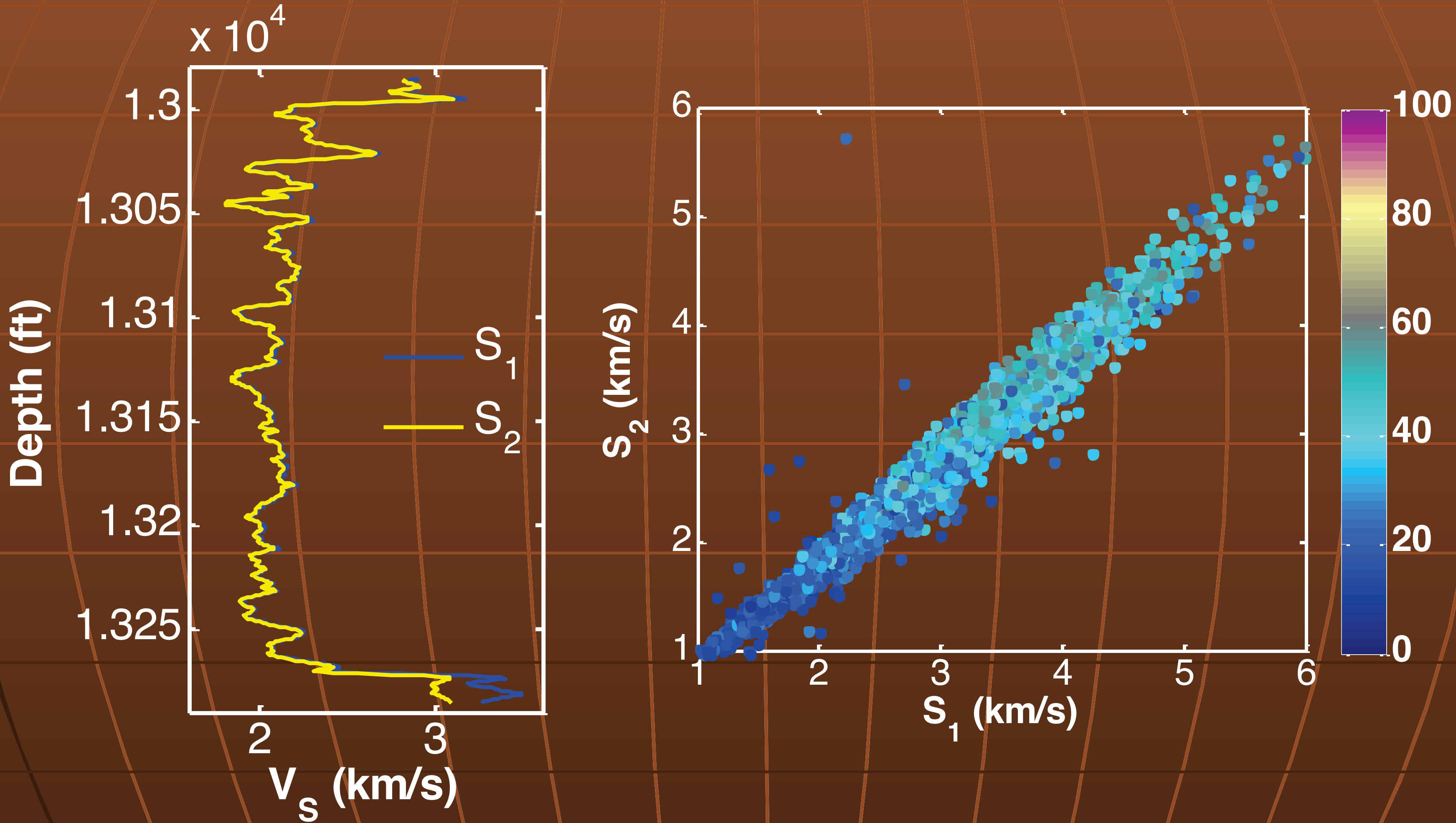

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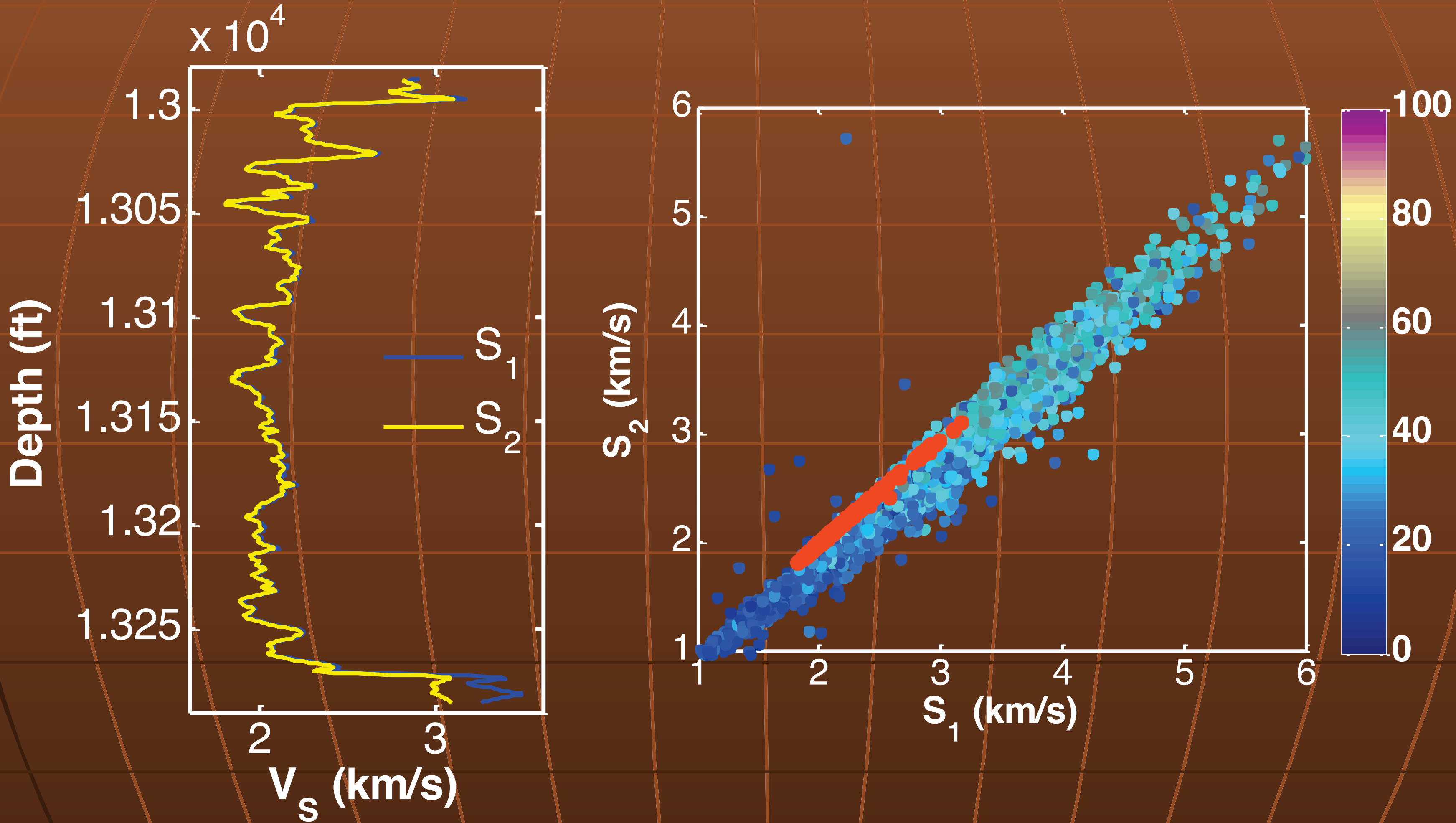

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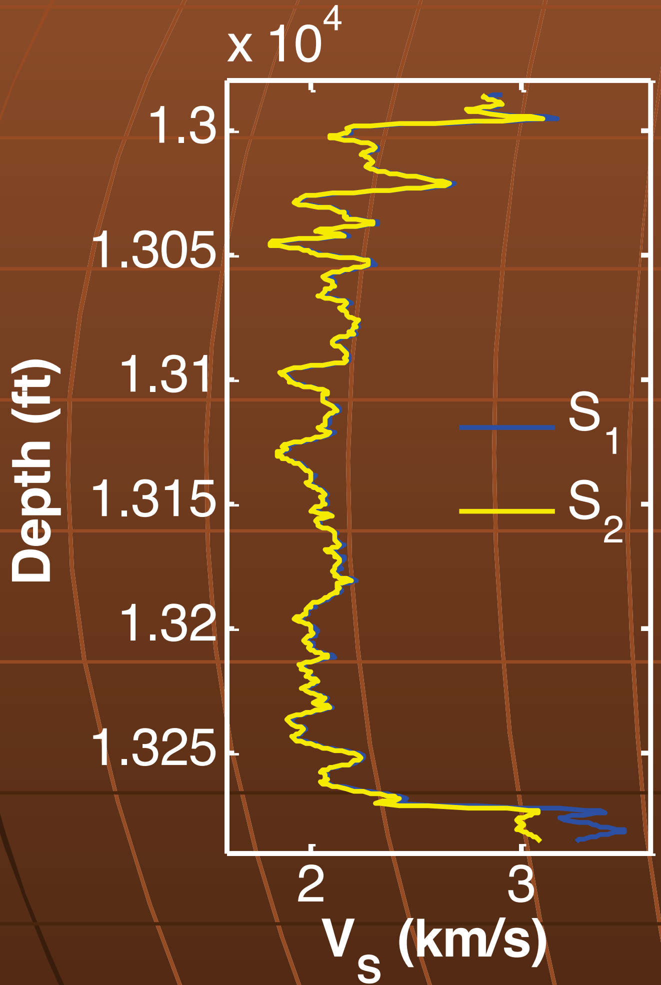

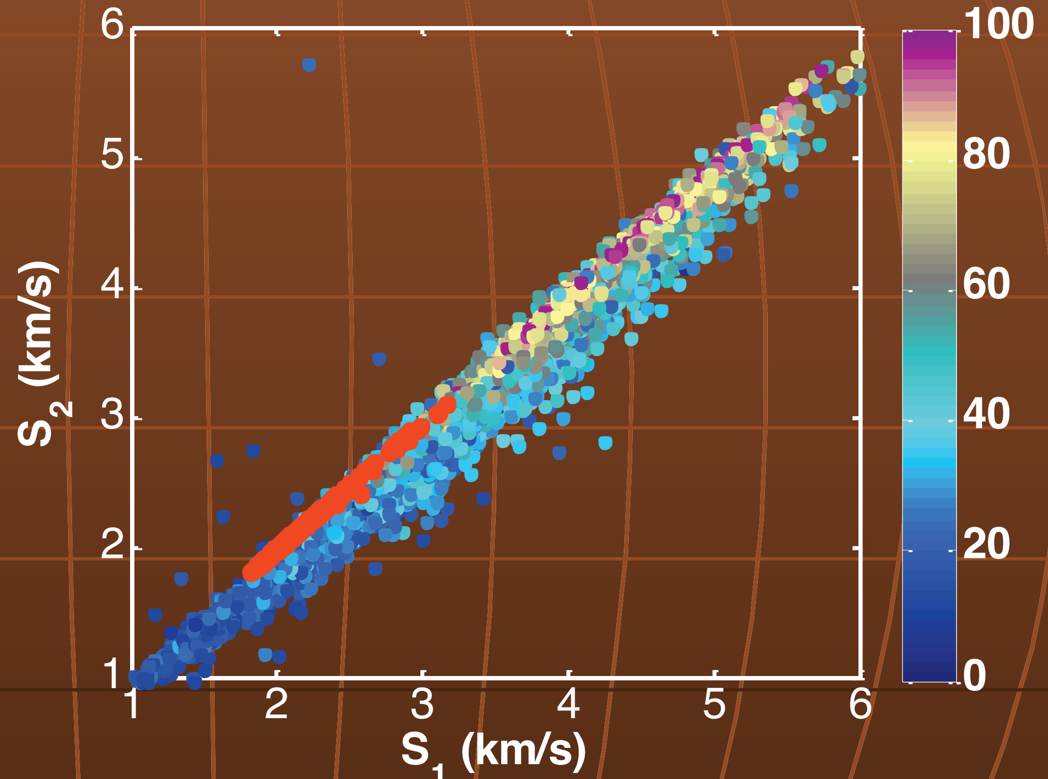

Conclusions

Modeling approaches indicate that a few percent cement can overprint the expected HTI

If cast properly, currently available effective medium models can be used to model more than one set of fractures (where the fractures are aligned)

Limitations of effective medium models start to show when applying to these potential situations

We will begin to verify these effective medium modeling approaches to advanced numerical wave propagation methods (e.g., discontinuous Galerkin)

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