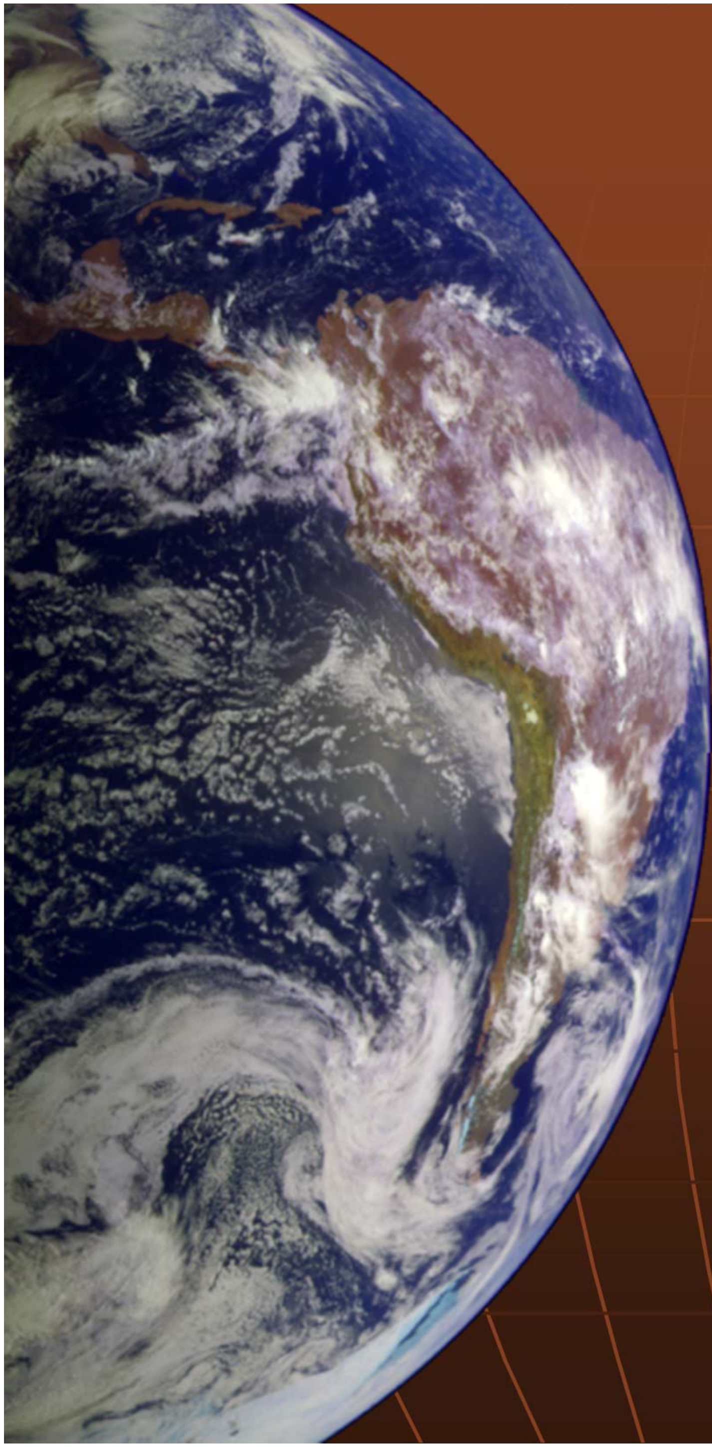


Greedy Annealed Importance Sampling for Seismic Inversion

Yang Xue

Advisor: Dr. Mrinal Sen

UT-AUSTIN EDGER FORUM

THE UNIVERSITY OF TEXAS AT AUSTIN
JACKSON
SCHOOL OF GEOSCIENCES

Overview

- Motivation
  - Seismic Inversion in Bayesian Frame
- Markov Chain Monte Carlo(MCMC)
  - Multiple Very Fast Simulated Annealing (VFSA)
  - Greedy Importance Sampling (GIS)
- Example of PP prestack inversion
- Comparison of different methods
- Conclusion

Motivation – Seismic Inversion

- Goal: simulate a log of rock properties as a function of two-way vertical travel time or depth
- Problem: limited wells
- Purpose of seismic inversion: derive a pseudo-log to fill in the gaps between wells

Geophysical inversion has non unique solutions

➡ Derive models with uncertainties

THE UNIVERSITY OF TEXAS AT AUSTIN
JACKSON
SCHOOL OF GEOSCIENCES

HOW?

Bayes Theorem

Posterior pdf

Likelihood

Prior pdf

Bayes theorem: $\sigma(\mathbf{m}|\mathbf{d}) \propto l(\mathbf{d}|\mathbf{m})\, p(\mathbf{m})$

$$E(\mathbf{m}) = (\mathbf{d} - \mathrm{g}(\mathbf{m}))^T \, \mathbf{C}_D^{-1} (\mathbf{d} - \mathrm{g}(\mathbf{m}))$$

$$l(\mathbf{d}|\mathbf{m}) \propto \exp(-E(\mathbf{m}))$$

If prior distribution is uniform:

$$\sigma(m \,|\, d) \propto \exp\left[-(\mathbf{d} - \mathrm{g}(\mathbf{m}))^T \, \mathbf{C}_D^{-1} (\mathbf{d} - \mathrm{g}(\mathbf{m}))\right]$$

MCMC methods

Draw samples from Prior (uniform) distribution → Markov Chain → Samples in Posterior distribution

$$\langle \mathbf{m} \rangle = \sum \mathbf{m}\, \sigma(\mathbf{m})$$

- Importance Sampling
  - o Metropolis-Hastings/Gibbs' sampler
- Multiple realization of VFSA
- Greedy Importance Sampling
  - o Greedy search for the important region of posterior distribution

Very Fast Simulated Annealing

- Optimization method searches for the global minimum error or maximum a posteriori (MAP)
- Algorithm
  - o Loop over temperature (cooling down)
    - Loop over random moves
    - Update model parameter
    - Forward modeling
    - Calculate error function and acceptance probability
- Estimate models by weighted summation of all the accepted models and their uncertainties

(Sen, M. K., and P. L. Stoffa, 1995, Global optimization methods in geophysical inversion)

THE UNIVERSITY OF TEXAS AT AUSTIN
JACKSON
SCHOOL OF GEOSCIENCES

Greedy Importance Sampling Motivation

- Importance sampling:
  - o Draw samples from prior distribution Q, assign weights $w_i = Q / P$, estimate expectation: $E_{P(x)} f(x) = \sum_{i=1}^{i=n} f(x_i) w(x_i) / n$
- Problem:
  - o *Q* misses high probability regions of *P*
- Methodology:

  Search for significant region in posterior distribution *P* with sampling from the prior distribution *Q*
- Advantages:
  - o maintain unbiasedness, minimize variance

Greedy importance sampling flowchart

$(x_1,..x_i.,x_n)$ Q

$$A_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,m})$$

Compute successors by searching for local maximum of $|f(x)P(x)|$

$$D_i = (x_{i,-m+1}, .., x_{i,-1}, x_{i,0})$$

Compute predecessors by searching for local minimum of $|f(x)P(x)|$

final samples

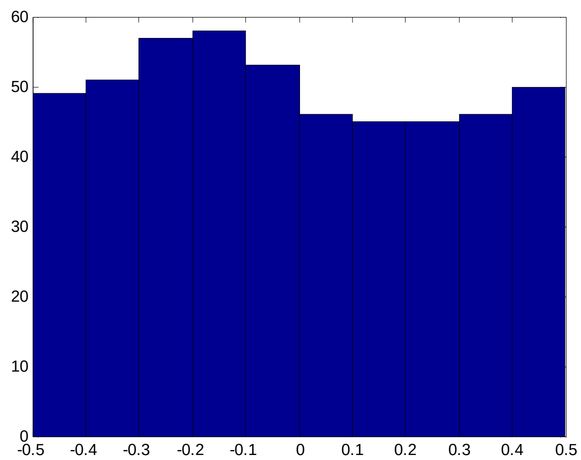
$$x_{1,1}, \ldots, x_{1,m}, x_{2,1}, \ldots, x_{2,m}, \ldots x_{n,1}, \ldots, x_{n,m}$$

$$\omega(x_{i,j}) = P(x_{i,j}) / \sum_{k=j-m+1}^{j} Q(x_{i,k})$$

$$E_{P(x)} f(x) = \frac{1}{n} \sum_{i=1}^{i=n} \sum_{k=1}^{m} f(x_{i,k}) w_i(x_k)$$

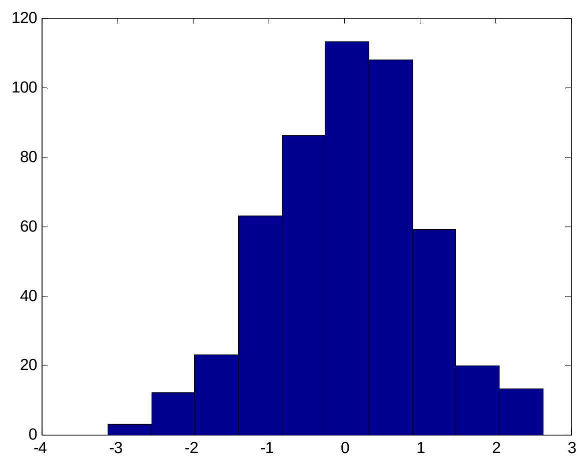
(Dale Shuurmans and Finnegan Southey)

Example of GIS

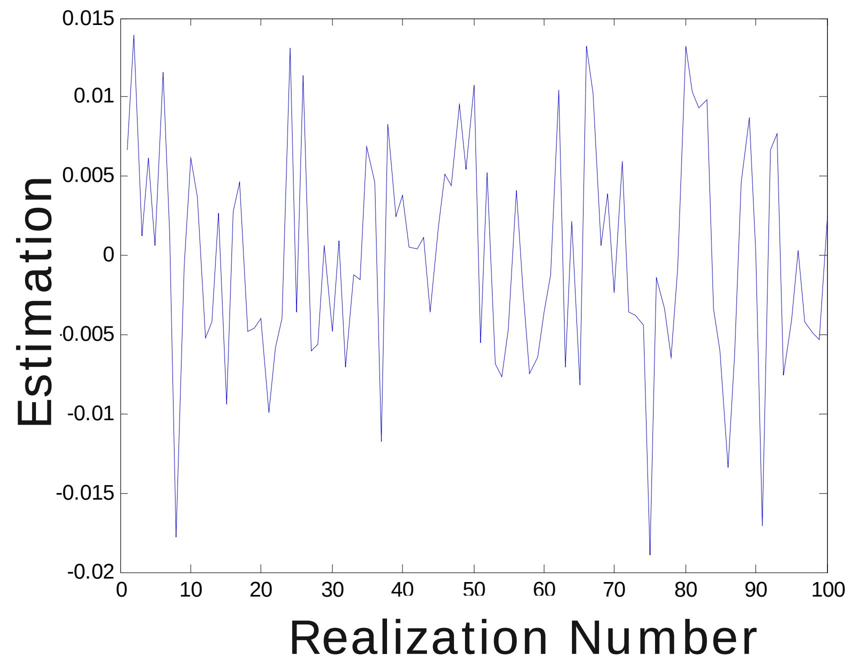


Q (prior)

f(x)=x



P(posterior)



Real. Nr.	Deviation
10	-1.719e-4
100	3.1221e-6

Greedy annealed importance sampling flowchart

$(T_1,..T_i.,T_n)_{\text{uniform}}$

$A_i = (T_{i,1}, T_{i,2}, \ldots, T_{i,m})$

Run VFSA to generate Vp, Vs, ρ and compute successors by searching for local maxi. of $|mP(m)|$

$D_i = (T_{i,-m+1}, .., T_{i,-1}, T_{i,0})$

Run VFSA to generate Vp, Vs, ρ and compute predecessors by searching for local min. of $|mP(m)|$

final samples

$m_{1,1}, \ldots, m_{1,m}, m_{2,1}, \ldots, m_{2,m}, \ldots m_{n,1}, \ldots, m_{n,m}$

$$P(m_{i,j}) = e^{-|E(T_i)|/T_i} / \sum_{i=1}^{m} e^{-|E(T_i)|/T_i}$$

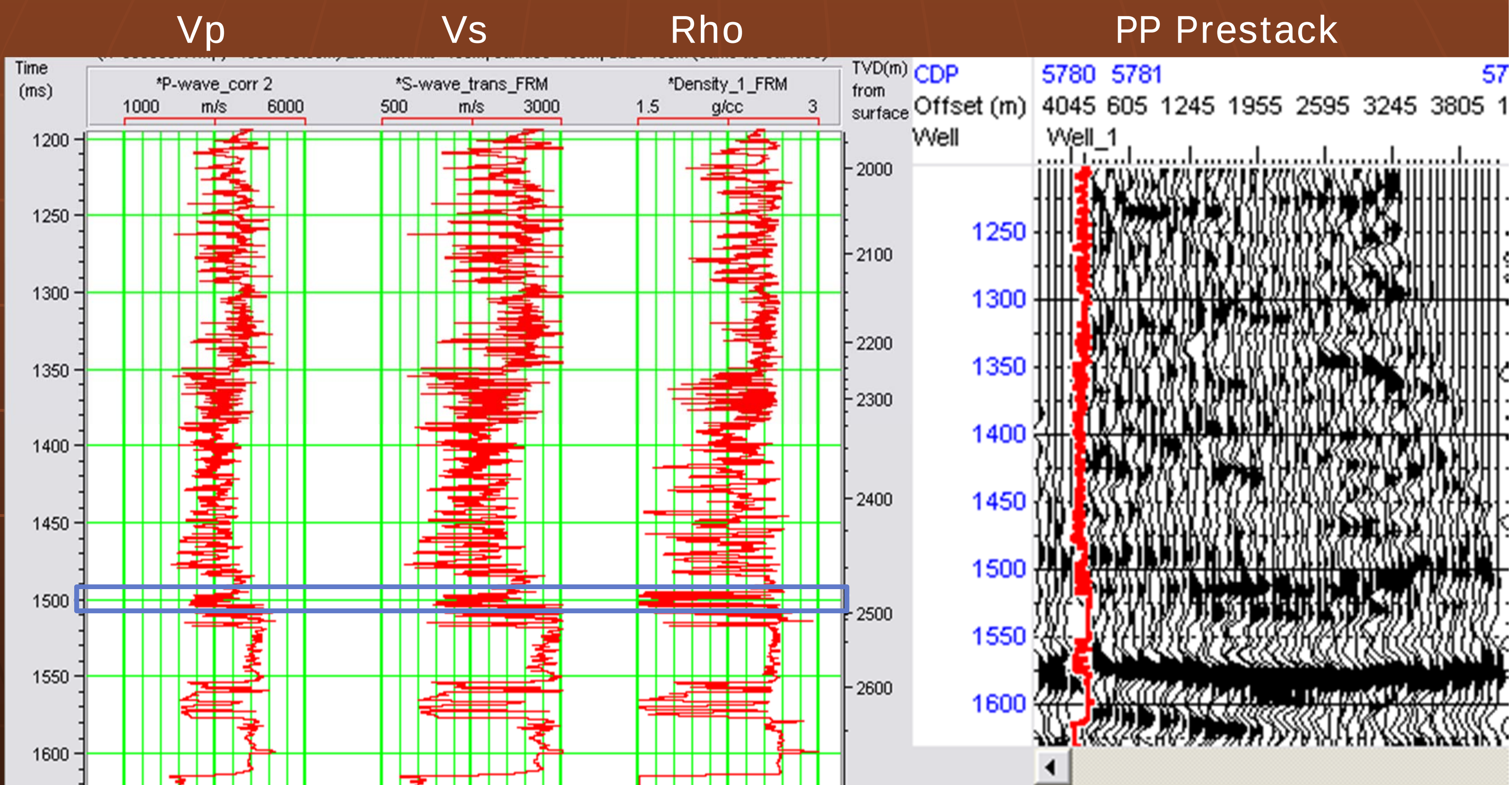
$$\omega(m_{i,j}) = P(m_{i,j}) / \sum_{k=j-m+1}^{j} Q(m_{i,k})$$

$$E_{P(m)} m = \frac{1}{n} \sum_{i=1}^{i=n} \sum_{j=1}^{m} m_{i,j} \omega_{i,j}$$

$m : Vp, Vs, \rho$

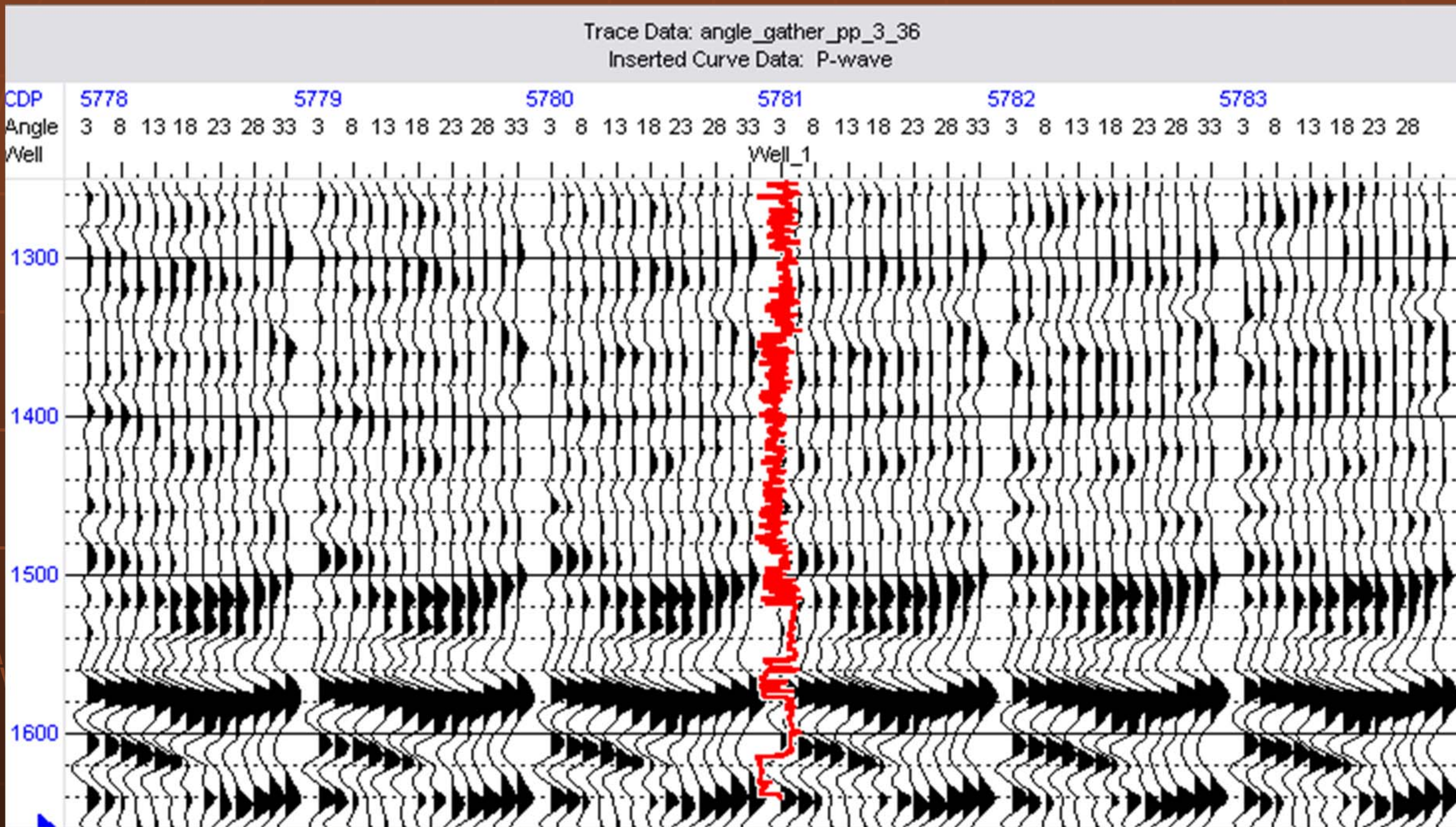
THE UNIVERSITY OF TEXAS AT AUSTIN
JACKSON
SCHOOL OF GEOSCIENCES

Example: PP prestack inversion



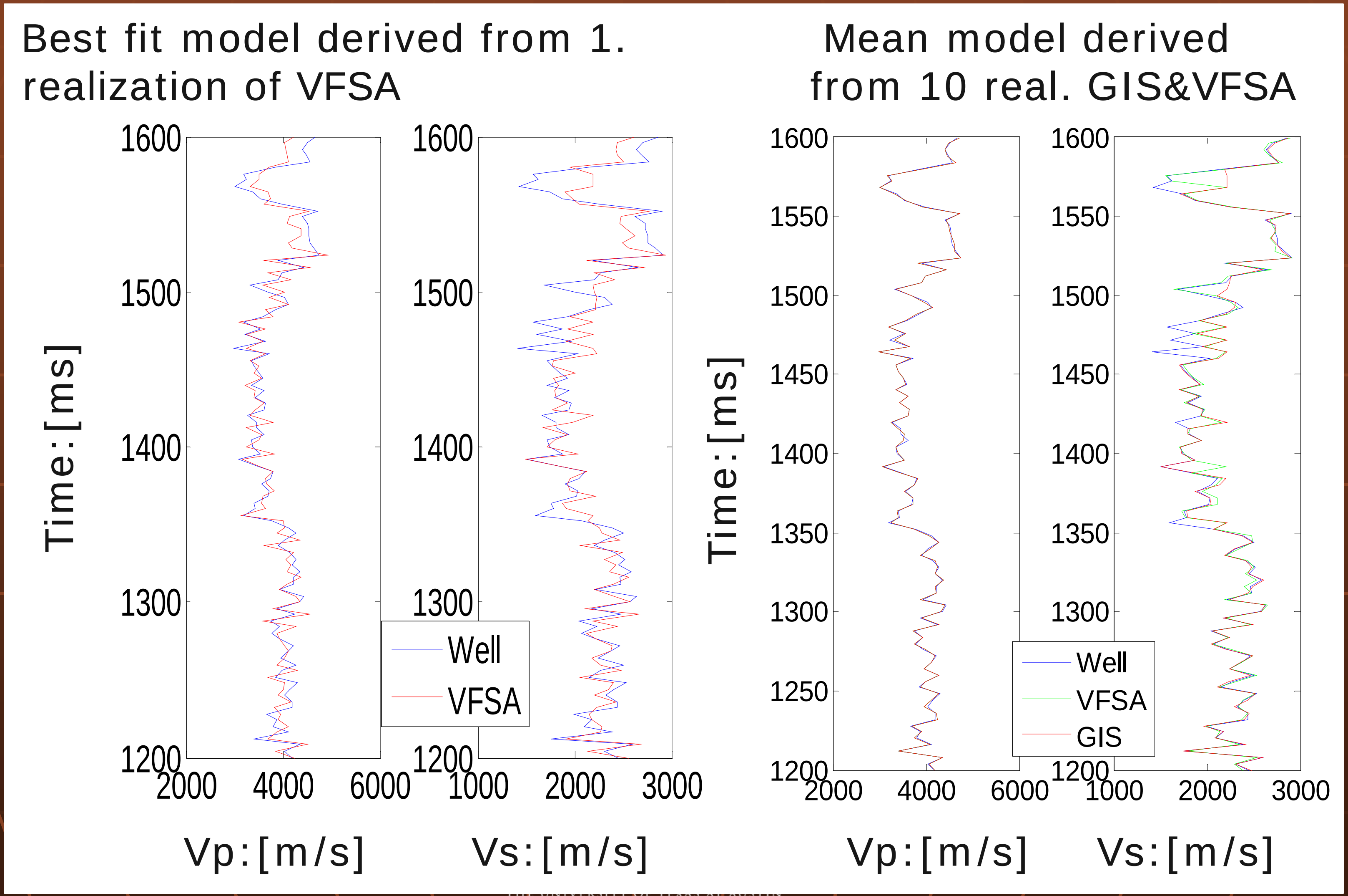
THE UNIVERSITY OF TEXAS AT AUSTIN
JACKSON
SCHOOL OF GEOSCIENCES

Example: PP prestack inversion PP angle gather: 3-36degree

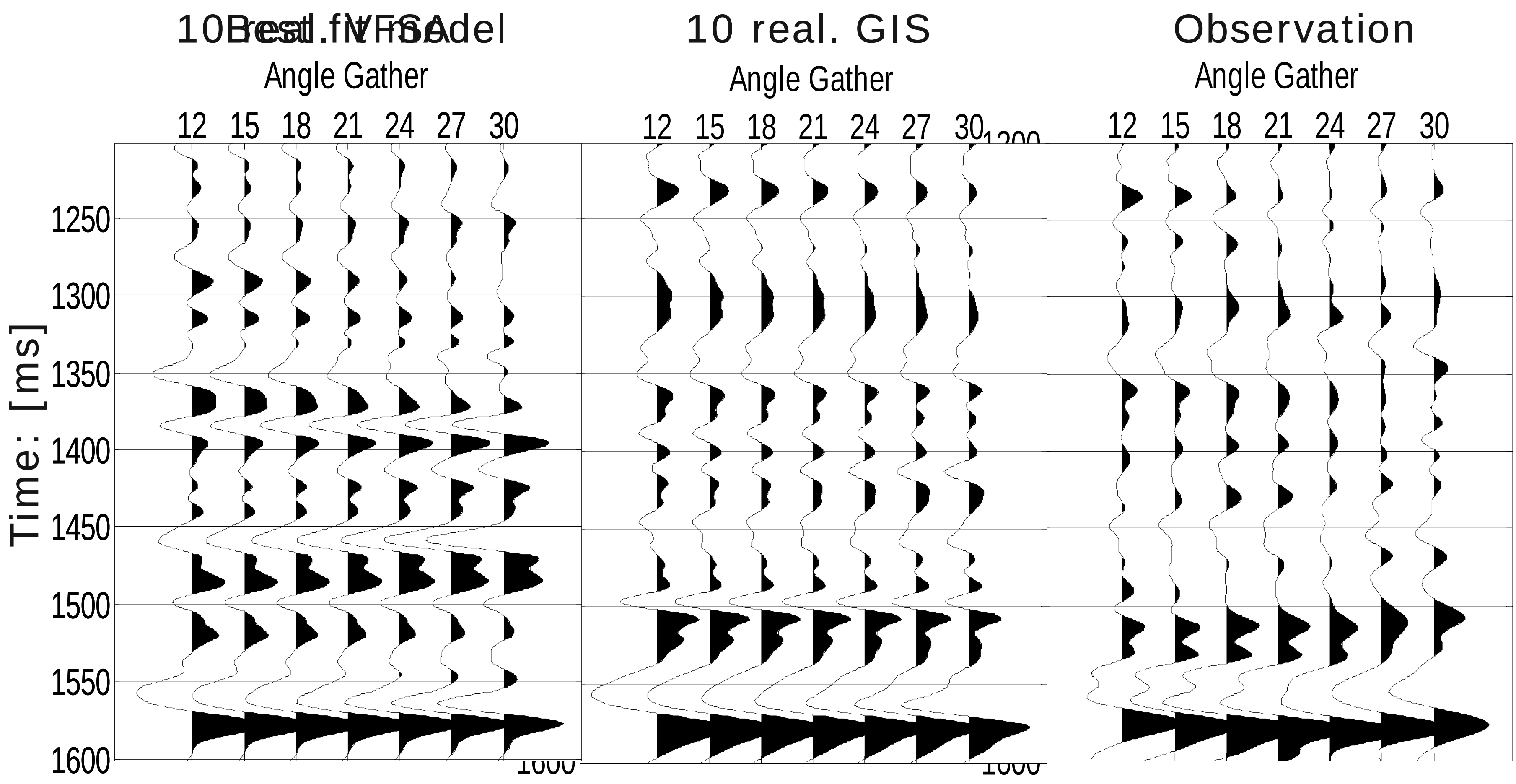


THE UNIVERSITY OF TEXAS AT AUSTIN
JACKSON
SCHOOL OF GEOSCIENCES

Comparison of velocity models

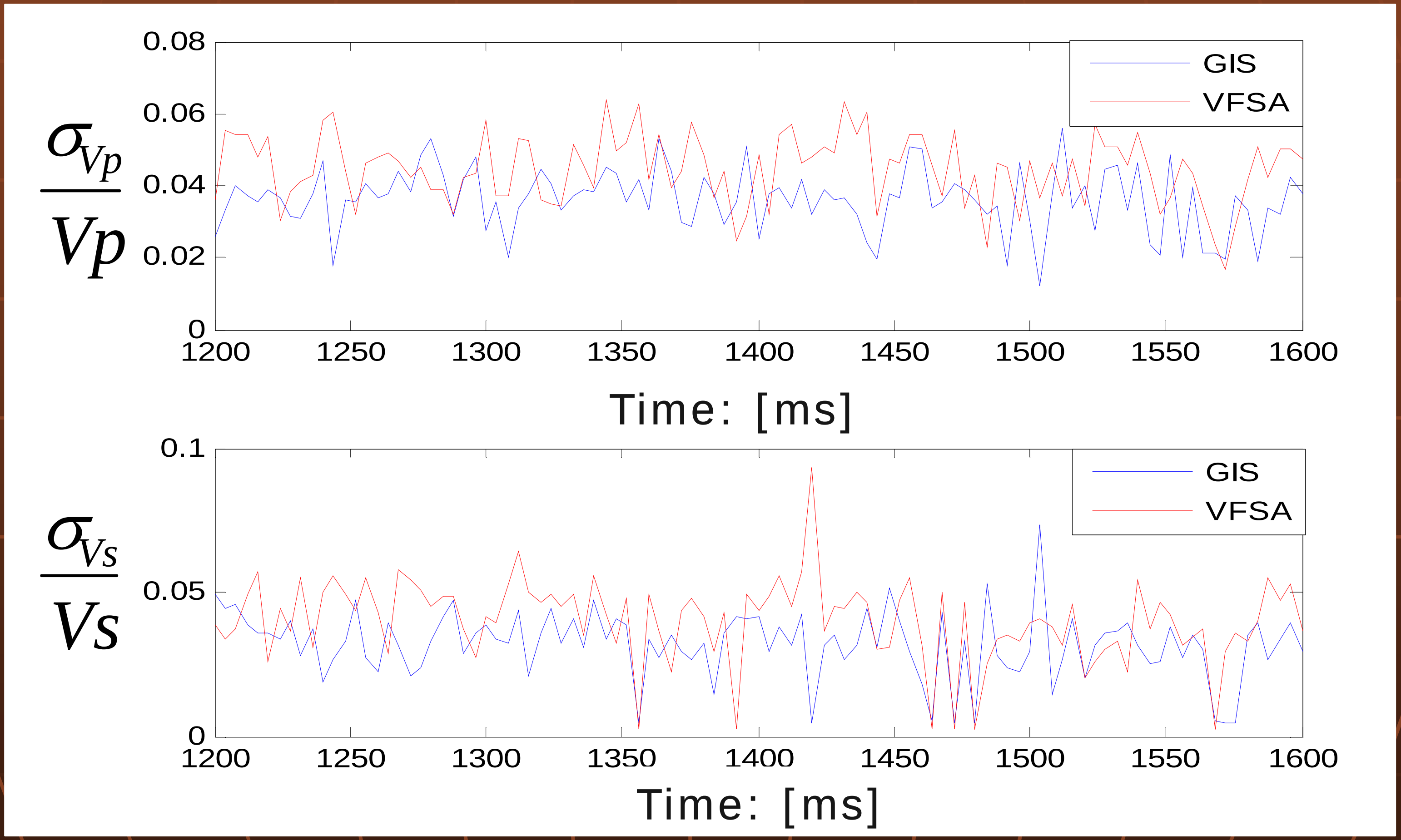


Comparison of seismic traces



23-Feb-2011 17:41:52

Comparison of standard deviation of model parameters



THE UNIVERSITY OF TEXAS AT AUSTIN
JACKSON
SCHOOL OF GEOSCIENCES

Summary

- Solution of a geophysical inverse problem involves drawing samples from the posterior probability density (PPD) which is complicated and cannot, in general, be described analytically.
- MCMC methods based on Metropolis-Hastings Gibbs's sampler is theoretically correct (gives unbiased estimates) but it computationally very slow.
- Greedy importance sampling (GIS) also produces unbiased estimates and is faster than a standard MCMC

Summary

- Multiple VFSA is an approximate method which is very fast. The estimates are, however, biased. Posterior variance is typically under-estimated.
- We have developed a new technique that combines VFSA with GIS which produces unbiased estimates faster than a GIS.

Thank you for your attention!

Appreciation to:

Prof. Dr. Mrinal Sen

Dr. zhiwen Deng

Thomas Hess

Son Phan

Rui Zhang

THE UNIVERSITY OF TEXAS AT AUSTIN
JACKSON
SCHOOL OF GEOSCIENCES

Special Thanks to our Sponsors

UT-AUSTIN EDGER FORUM

Anadarko

bp

BRIGHAM Exploration Company

CIMAREX

devon

ConocoPhillips

CNPC BGP

Chevron Human Energy

drillinginfo® Open To Explore™

HALLIBURTON | Landmark Software & Services

DAWSON® Dawson Geophysical Company