Reflection from a fractured porous medium

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Questions

- How do we verify effective media models?

 [Part I]

- Given effective stiffness tensors, how do we compute reflection coefficients?

[Part II]

Background

- How do we compute reflection coefficients?

Boundary Conditions

Liquid/Solid Interface

We know $\boldsymbol{\tau}_{\text{liquid}} \cdot \hat{z} = \boldsymbol{\tau}_{\text{solid}} \cdot \hat{z}$ The additional conidition we impose is "anti-cavitation condition" which is $\hat{z} \cdot \boldsymbol{u}_{\text{liquid}} = \hat{z} \cdot \boldsymbol{u}_{\text{solid}}$, i.e. the normal components of the displacement must be continuous. Notice that all of the displacement components need not be continuous across the interface because the liquid is free to move sideways

$$(\tau_{zz})_{\text{solid}} = (\tau_{zz})_{\text{liquid}}$$

$$(\tau_{zx})_{\text{solid}} = (\tau_{zx})_{\text{liquid}} = 0$$

$$(\tau_{zy})_{\text{solid}} = (\tau_{zy})_{\text{liquid}} = 0$$

Boundary Conditions

<u>Solid/Solid Interface</u>

$$\boldsymbol{\tau}_1 \cdot \hat{z} = \boldsymbol{\tau}_2 \cdot \hat{z}$$

$$\boldsymbol{u}_1 \cdot \hat{z} = \boldsymbol{u}_2 \cdot \hat{z}$$

<u>Solid/Solid (Welded Contact)</u>

$$\boldsymbol{\tau}_1 \cdot \hat{z} = \boldsymbol{\tau}_2 \cdot \hat{z}$$

$$\boldsymbol{u}_1 = \boldsymbol{u}_2$$

All three components of displacement are continuous since the solid being in welded contact is not allowed to move sideways. In other words the vector is continuous.

Reflection Coefficients

Aki and Richards (2002).

Steps

- incident plane waves have unit amplitude; the reflected and transmitted waves have the amplitudes of reflection/transmission coefficients
- express displacements and stress components in terms of these plane wave potentials
- Apply boundary condition at the boundary
- Solve the resulting system of equations for the R/T coefficients

Faults/Fractures

- Linear Slip Model (Schoenberg)
- Stresses are continuous – displacement discontinuous
- The resulting reflection coefficients are frequency dependent
- Observation of a phase shift even at normal incidence

[van der Neut, Sen and Wapanear 2007]

Part I

Seismic Wave Propagation in Fractured Media

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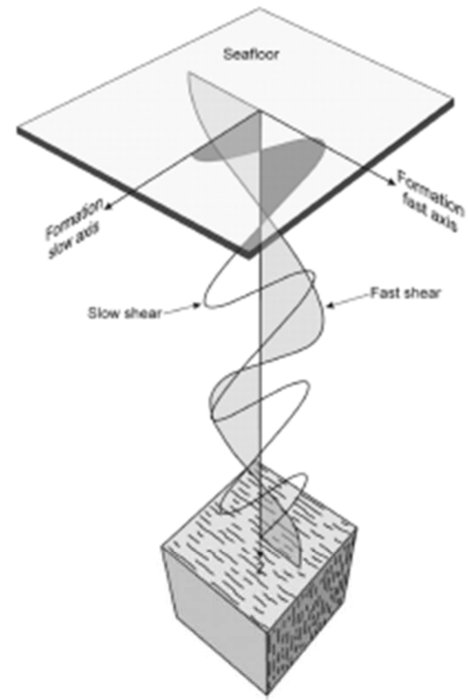
Motivation

- Fractures are a **common feature in the subsurface**,
- Observed in many scales, from faults to micro-cracks,
- Parallel micro-cracks introduce **seismic anisotropy** (Schoenberg & Douma, 1988),
- Characterization of the orientation and density of fractures has important practical applications (Sayers, 2007).

- Two approaches to incorporate the effects of fractures in wave propagation:
  1. Using equivalent media theories,
  2. Simulating the fractures in a numerical scheme.



(Barton, 2007)

Goals

1. Develop a numerical approach to incorporate fractures in wave-propagation simulations,
2. Validate the Equivalent Media Theories,
3. Investigate numerically the sensitivity of the data to the fracture parameters.

Equivalent Media Theories

- Equivalent Media Theories predict the **effective elastic properties** of fractured media given some fracture parameters.
- Common assumptions:
  - Idealized crack shape,
  - small aspect ratio and crack density compared to wavelength,
  - Cracks are isolated with respect to fluid flow.
- Examples of Effective Media Theories:
  - Kuster-Toksöz,
  - Differential Effective Medium,
  - Hudson,
  - Eshelby-Cheng.

(Mavko et al., 1998; Saenger et al., 2004, and references therein)

Numerical Approaches

- Approaches that have been proposed in the literature:
  - Use locally an effective medium (Vlastos et al., 2003),
  - Incorporate locally a low velocity and low density inclusion into a finite difference scheme (Saenger & Shapiro, 2002; Saenger et al., 2004), and
  - Explicitly use a displacement discontinuity condition using the linear-slip model (Zhang, 2005; Zhang & Gao, 2009).
- The advantage: they require few assumptions and therefore they have a broad applicability and are useful to validate the equivalent medium theories.
- Approaches based on the linear-slip model require the least number of assumptions.

The Linear Slip Model

Linear-Slip Model (LSM): Prescribes a linear relation between the traction vector and jump in the displacement:

$$[\boldsymbol{u}] = \boldsymbol{Z}\boldsymbol{\tau}, \tag{1}$$

where $[\boldsymbol{u}]$ is the jump of the displacement, $\boldsymbol{\tau}$ is the traction vector at the fracture and $\boldsymbol{Z}$ is the fracture compliance matrix. For a fracture with rotational symmetry about the normal, the fracture compliance matrix is given by (Schoenberg & Douma, 1988; Zhang & Gao, 2009)

$$Z_{ij} = Z_N n_i n_j + Z_T(\delta_{ij} - n_i n_j), \tag{2}$$

where Z_T and Z_N are the tangential and normal components of the compliance matrix.

Discontinuous Galerkin Method

- The Discontinuous Galerkin Method (DGM) is a generalization of FEM that allows for the basis functions to be discontinuous at the element interfaces.
- IP-DGM: Interior-penalty formulation
  - SIPG: Symmetric Interior Penalty Galerkin (Darlow, 1980),
  - NIPG: Non-symmetric (Rivière & Wheeler, 2001),
  - IIPG: Incomplete (Dawson et al., 2004).
- Advantages
  - it can accommodate discontinuities in the wave field,
  - it can be energy conservative,
  - it can handle more general meshes, and
  - it is suitable for local time stepping and parallel implementations.

Accuracy and Stability of DGM

- Grid dispersion and stability analyzed in De Basabe et al. (2008) and De Basabe & Sen (2010).
- **Superconvergence** of the grid-dispersion error with respect to the sampling ratio for the symmetric formulation and nodal basis functions,
- The numerical **anisotropy** is negligible for basis of degree 4 or greater,
- **Stability** condition in 2D given by

$$\frac{\alpha \Delta t}{\Delta x} \leq 0.25,$$

where Δx is the smallest spatial increment, Δt is the size of the time step and α is the largest P-wave velocity.

Interior-Penalty Weak Formulation

Find $\boldsymbol{u} \in \boldsymbol{X}^D$ such that for all $\boldsymbol{v} \in \boldsymbol{X}^D$

$$\sum_{E \in \Omega_h} \left((\rho \partial_{tt} \boldsymbol{u}, \boldsymbol{v})_E + \boldsymbol{B}_E(\boldsymbol{u}, \boldsymbol{v}) \right) + \sum_{\gamma \in \Gamma_h} \boldsymbol{J}_\gamma^c(\boldsymbol{u}, \boldsymbol{v}; S, R) = \sum_{E \in \Omega_h} (\boldsymbol{f}, \boldsymbol{v})_E$$

where $X^D = \left\{ \varphi \mid \varphi \in \boldsymbol{H}^1(E)\ \forall\, E \in \Omega_h,\ \varphi = 0 \text{ on } \Gamma_D \right\}$

$$\begin{aligned}
\boldsymbol{B}_E(\boldsymbol{u}, \boldsymbol{v}) &= \int_E \left(\lambda \partial_i u_i \partial_j v_j + \mu(\partial_j u_i + \partial_i u_j)\partial_j v_i \right) d\Omega, \\
\boldsymbol{J}_\gamma^c(\boldsymbol{u}, \boldsymbol{v}; S, R) &= -\int_\gamma \{\tau_i(\boldsymbol{u})\}[v_i]\, d\gamma + S \int_\gamma \{\tau_i(\boldsymbol{v})\}[u_i]\, d\gamma \\
&+ R \int_\gamma \{\lambda + 2\mu\}[u_i][v_i]\, d\gamma, \\
\tau_i(\boldsymbol{u}) &= \sigma_{ij}(\boldsymbol{u}) n_j = \lambda u_{k,k} n_i + \mu(u_{i,j} + u_{j,i}) n_j.
\end{aligned}$$

The parameter R is the penalty, and S is a parameter that takes the values $S = 0$ for IIPG, $S = -1$ for SIPG and $S = 1$ for NIPG.

Proposed Numerical Scheme

Find $\boldsymbol{u} \in \boldsymbol{X}^D$ such that for all $\boldsymbol{v} \in \boldsymbol{X}^D$

$$\sum_{E\in\Omega_h} \Big((\rho\partial_{tt}\boldsymbol{u}, \boldsymbol{v})_E + \boldsymbol{B}_E(\boldsymbol{u}, \boldsymbol{v}) \Big) + \sum_{\gamma\in\Gamma_c} \boldsymbol{J}^c_\gamma(\boldsymbol{u}, \boldsymbol{v}; S, R) + \sum_{\gamma\in\Gamma_f} \boldsymbol{J}^f_\gamma(\boldsymbol{u}, \boldsymbol{v}; S, R) = \sum_{E\in\Omega_h} (\boldsymbol{f}, \boldsymbol{v})_E$$

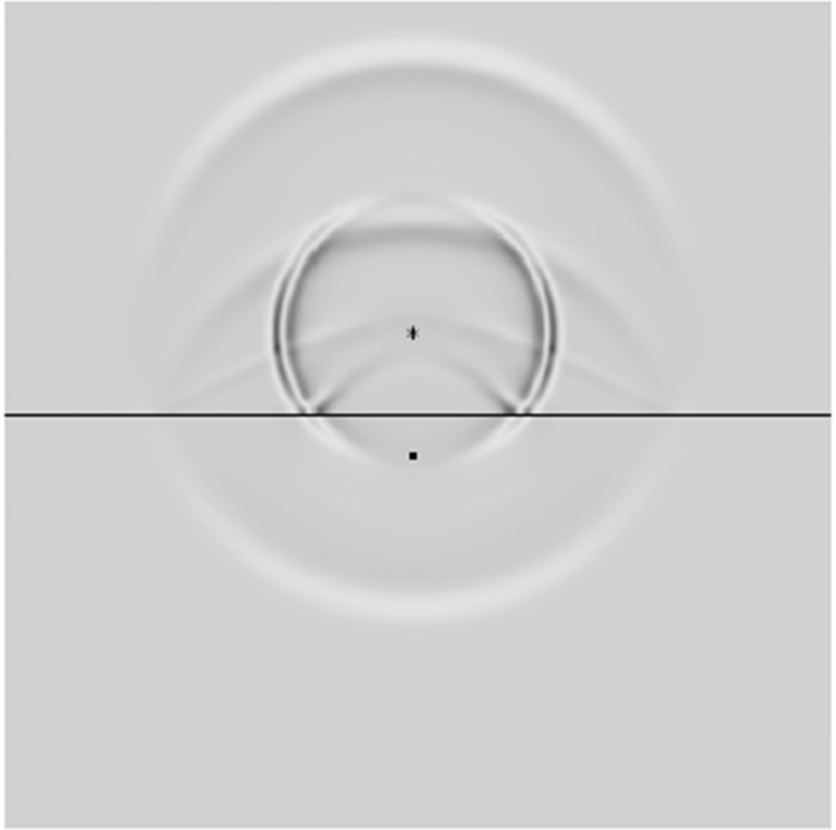
where $\Gamma_c \subset \Gamma_h$ is the subset of all faces where the displacement field is continuous, $\Gamma_f \subset \Gamma_h$ is the subset of faces that represent fractures, and

$$\boldsymbol{J}^f_\gamma(\boldsymbol{u}, \boldsymbol{v}) = \int_\gamma Z^{-1}_{ij} [u_j][v_i] \, d\gamma.$$

The linear slip condition is weakly imposed through this term.

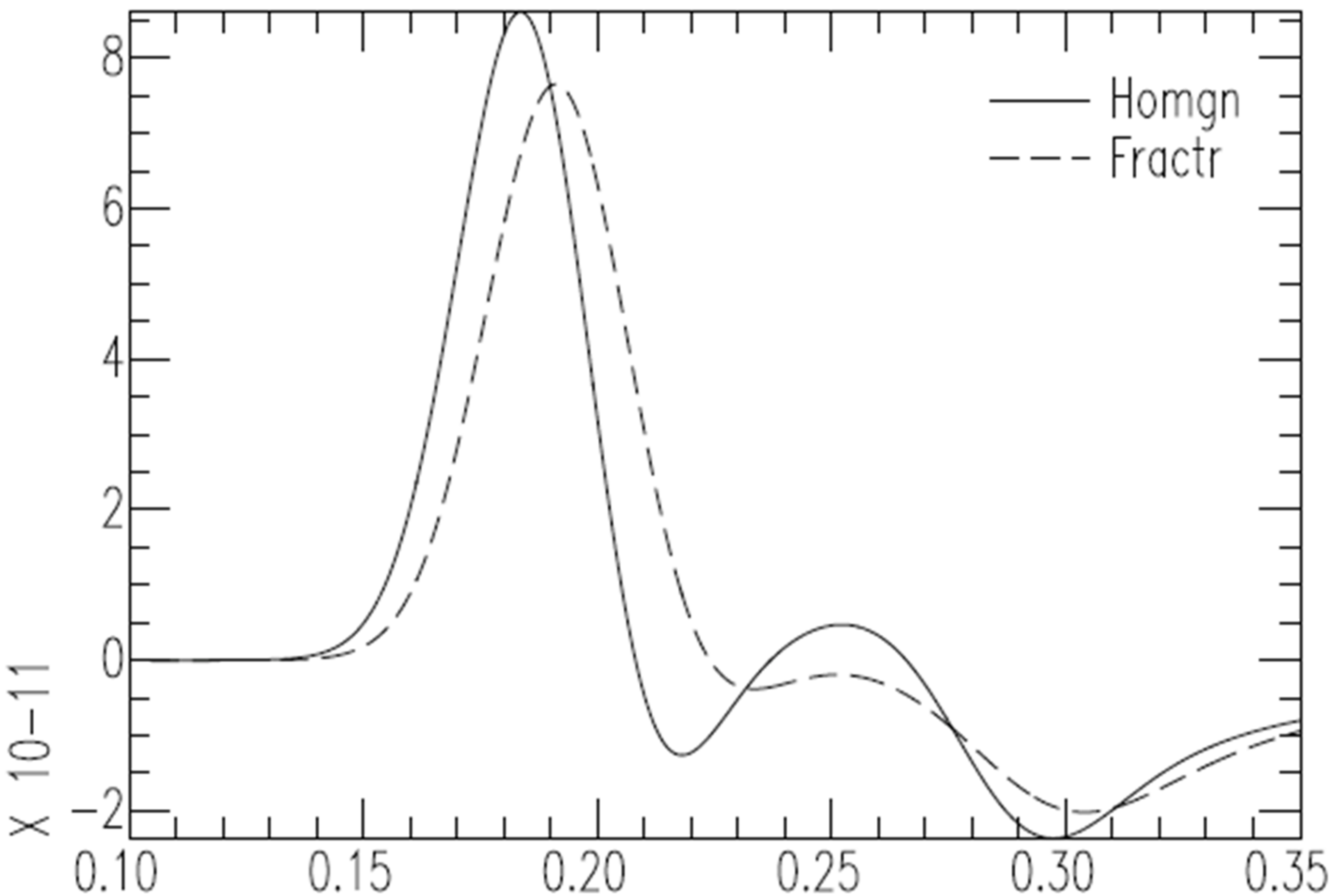
Results

- Domain: 1 Km$\times$ 1 Km,
- Point source at (0.5, 0.3),
- Horizontal fracture centered at (0.5, 0.5),
- $V_P = 3.31$ Km/s, $V_S = 1.62$ Km/s, $\rho = 2.5$ g/cm^3

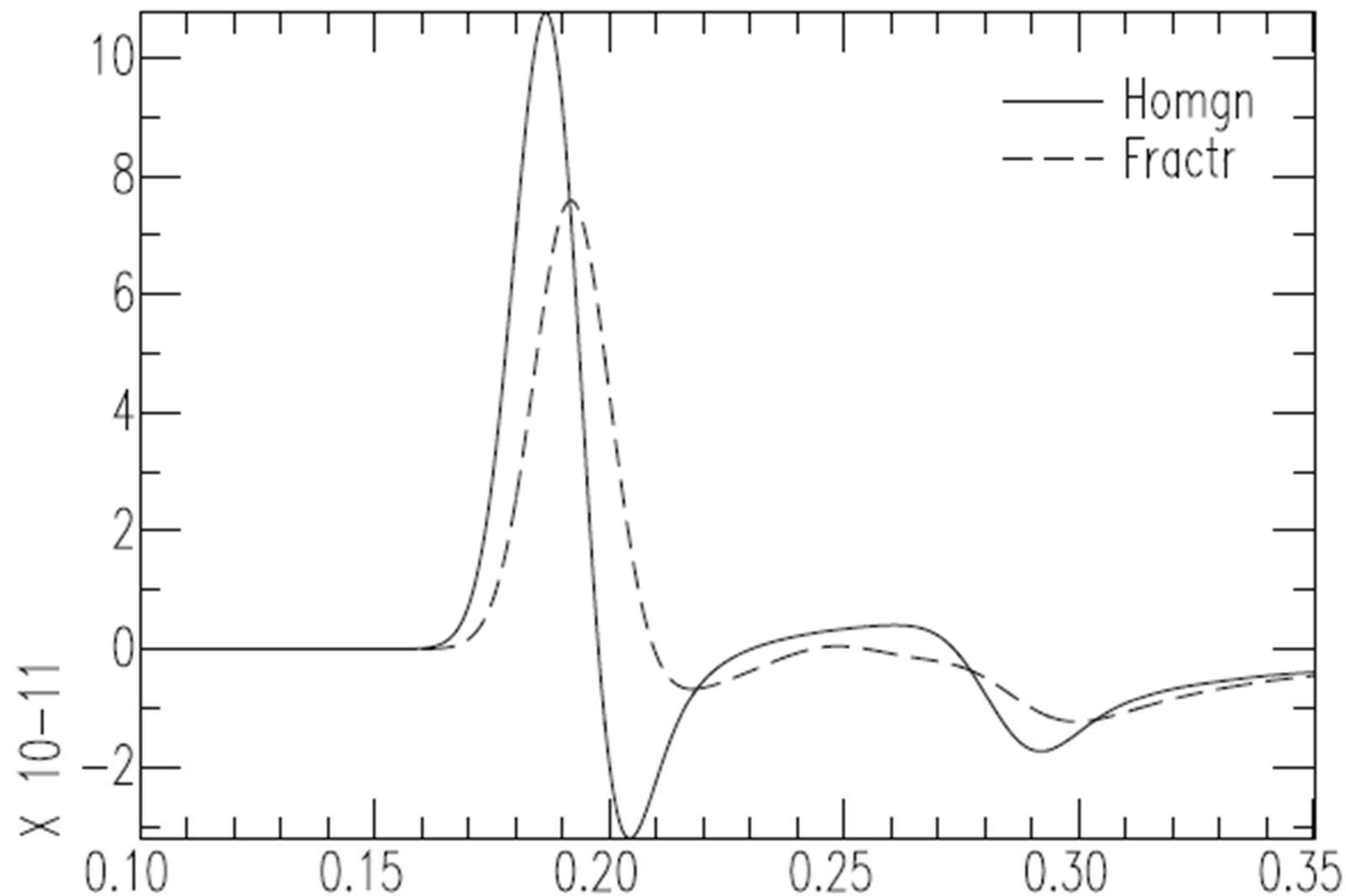


Results

Seismograms of the homogeneous and faulted models – 15 Hz source



Results
Seismograms of the homogeneous and faulted models – 30 Hz source



Conclusions

- The numerical results show that the discontinuity introduces a phase shift and that the reflection and transmission coefficients are frequency dependent, in good agreement with the analytic solutions given in van der Neut et al. (2008).

- The proposed method is not restricted to simplified models, it can be applied to 3D models and arbitrary geometries for fractures and media parameters.

Part II

- Fractured Porous media → anisotropic stiffness coefficients that are frequency dependent.
- Anisotropic Systems: homogeneous, stratified and laterally heterogeneous media
- SVD: phase velocity, degenerate eigenvalues
- Eigen vector matrix, anisotropic reflection coefficient.

Heterogeneous Anisotropic Earth Models

Linearized Momentum Equation $\rho\,\ddot{\mathbf{u}} = \nabla.\tau + \mathbf{f}$

Constitutive Relation $\boldsymbol{\tau} = \mathbf{C} : \boldsymbol{\varepsilon} = \mathbf{C} : \nabla\mathbf{u}$

Wave Equation $\rho\,\ddot{\mathbf{u}} = \nabla.\left(\mathbf{C} : \nabla\mathbf{u}\right) + \mathbf{f}$

Field Variables: 6 components of stress and 3 components of particle displacements

Earth Model Parameters: 21 elastic coefficients and density

Frequency dependent anisotropy

$$\tau_{ij} = c_{ijkl} * u_{k,l} \tag{1}$$

in which '*' denotes time convolution and repeated subscripts imply summation. The momentum equation is

$$\rho\, \partial_t^2 u_i = \tau_{ij,j} + f_i \tag{2}$$

in which ρ is mass density, t is time and $\mathbf{f}$ is body force per unit volume. To remove derivatives in x_1, x_2 and t from these equations we take a triple Fourier transform:

$$g(p_1, p_2, \omega) = \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \times \int_{-\infty}^{\infty} dt \exp\left[i\omega(t - p_1 x_1 - p_2 x_2)\right] g(x_1, x_2, t). \tag{3}$$

After some algebraic manipulations one then obtains the first order system in the form

$$\partial_3 \mathbf{b} = i\omega \mathbf{A}\mathbf{b} - \frac{i}{\omega}\begin{pmatrix} \mathbf{0} \\ \mathbf{f} \end{pmatrix} \tag{4}$$

$$\mathbf{b} = \begin{pmatrix} \mathbf{u} \\ \mathbf{t} \end{pmatrix} \tag{5}$$

is the vector of motions, $\mathbf{u} = [u_1, u_2, u_3]^T$, and scaled tractions, $t = (i/\omega)[\tau_{13}, \tau_{23}, \tau_{33}]^T$. One reason for scaling the tractions is that it makes the components of $\mathbf{A}$ real when the medium is lossless, which is useful in certains kinds of computations. The system matrix $\mathbf{A}$ has the form

$$\mathbf{A} = \begin{pmatrix} \mathbf{T} & \mathbf{C} \\ \mathbf{S} & \mathbf{T}^T \end{pmatrix}, \tag{6}$$

where $\mathbf{T}$, $\mathbf{S}$, and $\mathbf{C}$ are 3×3 submatrices and $\mathbf{C}$ and $\mathbf{S}$ are symmetric. Note that $\mathbf{A}$ has this same form and these same symmetries even if any or all of ω, p_1, p_2, or the c_{ijkl} are

Fryer and Frazer 1985

- **A is diagonalizable**
- Eigenvectors of **A** with different eigenvalues are orthogonal
- There is a matrix **D and a diagonal matrix Λ such that AD=DΛ**
- **The elements of Λ are the upgoing and downgoing vertical slownesses**

How do we diagonalize A?

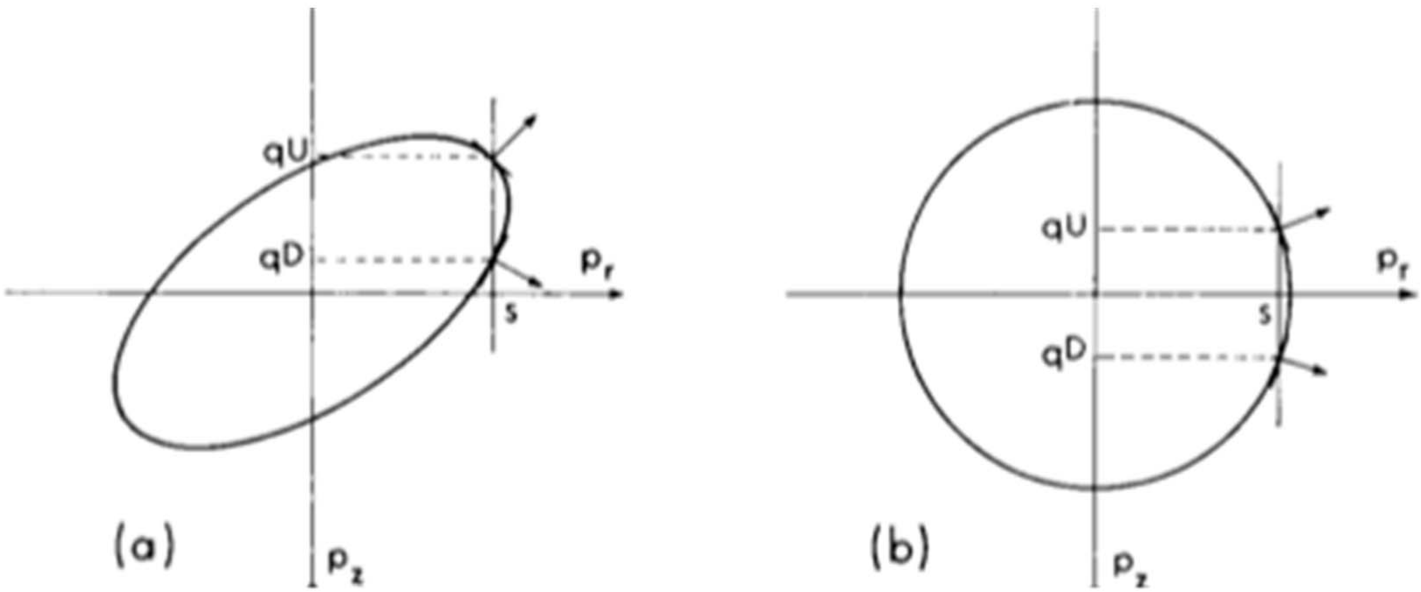
- It is a complex matrix and is NOT symmetric
- Apply a similarity transform to make **A** symmetric [Fryer and Frazer 1985]
- We can use Jacobi iteration to diagonalize a complex symmetric matrix

If **D** is the local eigenvector matrix of **A** then

$$\mathbf{D}^{-1}\mathbf{A}\mathbf{D} = \mathbf{\Lambda}$$

$$\mathbf{\Lambda} = \operatorname{diag}(q_P^{\mathrm{U}}, q_{S1}^{\mathrm{U}}, q_{S2}^{\mathrm{U}}, q_P^{\mathrm{D}}, q_{S1}^{\mathrm{D}}, q_{S2}^{\mathrm{D}})$$

$$\operatorname{Im}(q^{\mathrm{D}}) > 0 \qquad \text{and} \qquad \operatorname{Im}(q^{\mathrm{U}}) < 0.$$

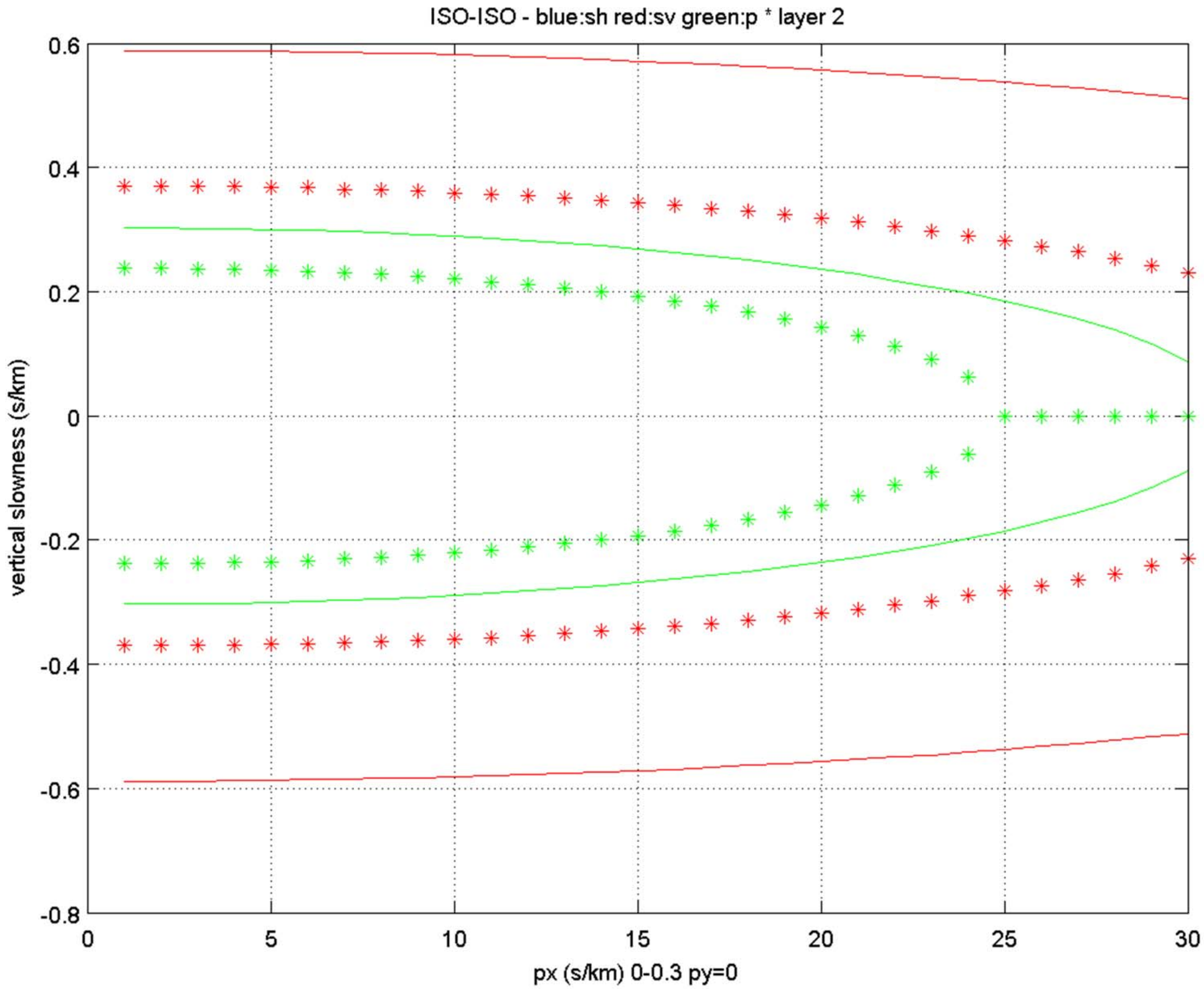


Fryer and Frazer 1985

Recipe for the calculation of R/T coefficient:

1. Form matrix $\boldsymbol{D}_{i-1}^{-1}$ for the medium $i-1$
2. Form matrix $\boldsymbol{D}_i$ for the medium i
3. Form $\boldsymbol{Q} = \boldsymbol{D}_{i-1}^{-1}\boldsymbol{D}_i$
4. Use submatrices of $\boldsymbol{Q}$ to form $\boldsymbol{R}$
5. Identify $r_{pp}, r_{ps}, r_{sp}, r_{ss}$, etc.

For anisotropic layers steps 3 and 5 need to be done numerically. For isotropic and transversely isotropic layers, step 3 can be carried out analytically.



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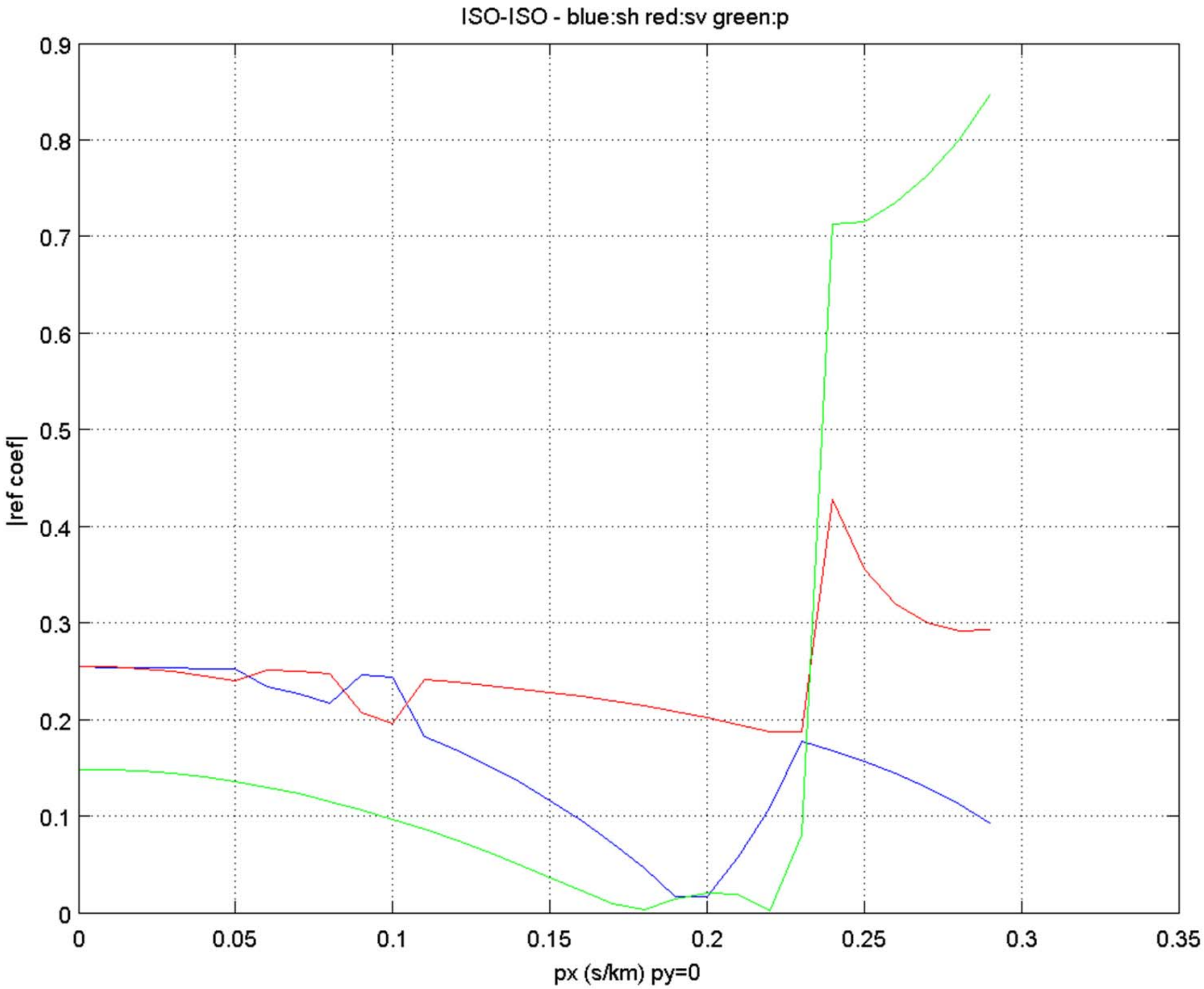
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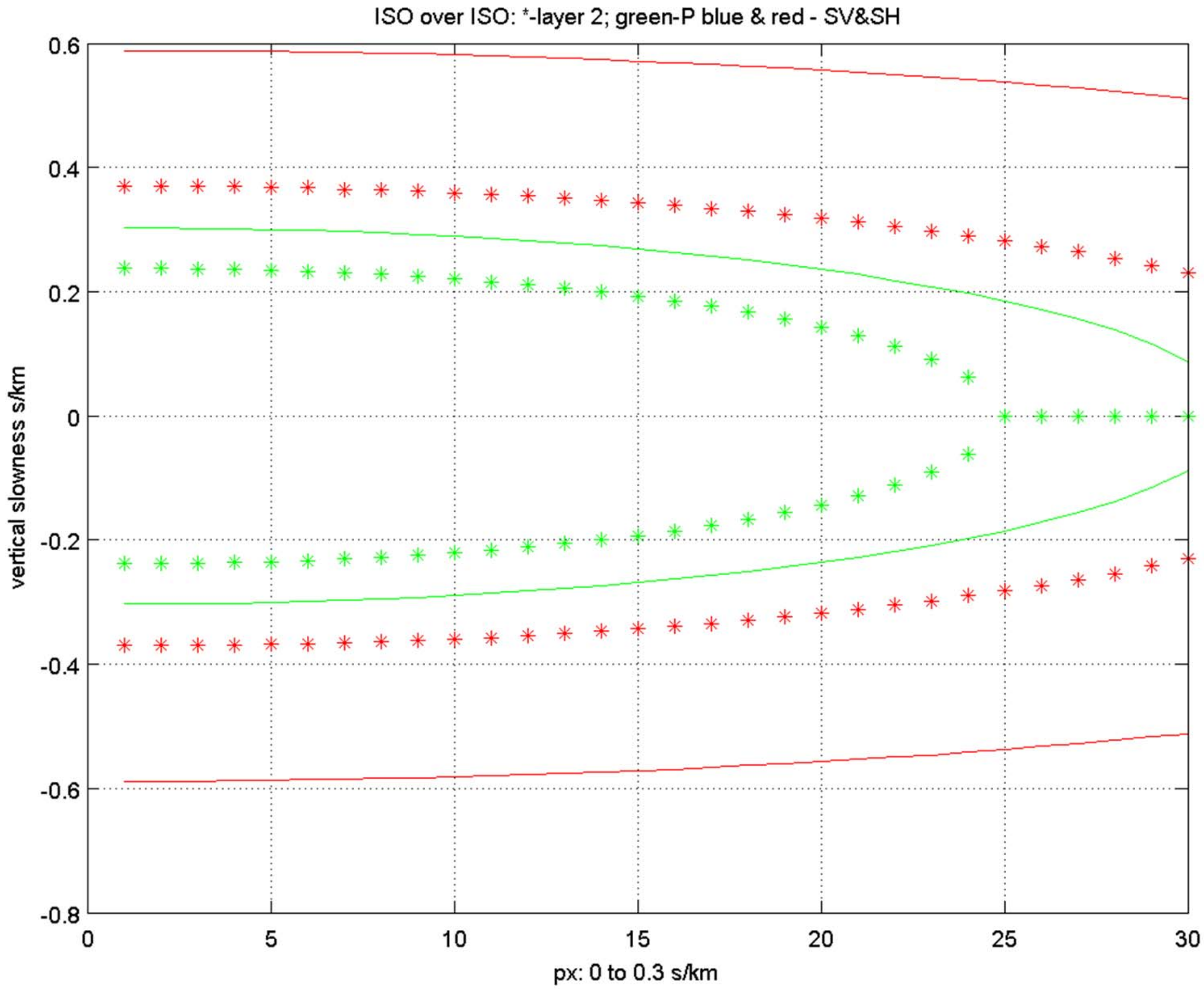
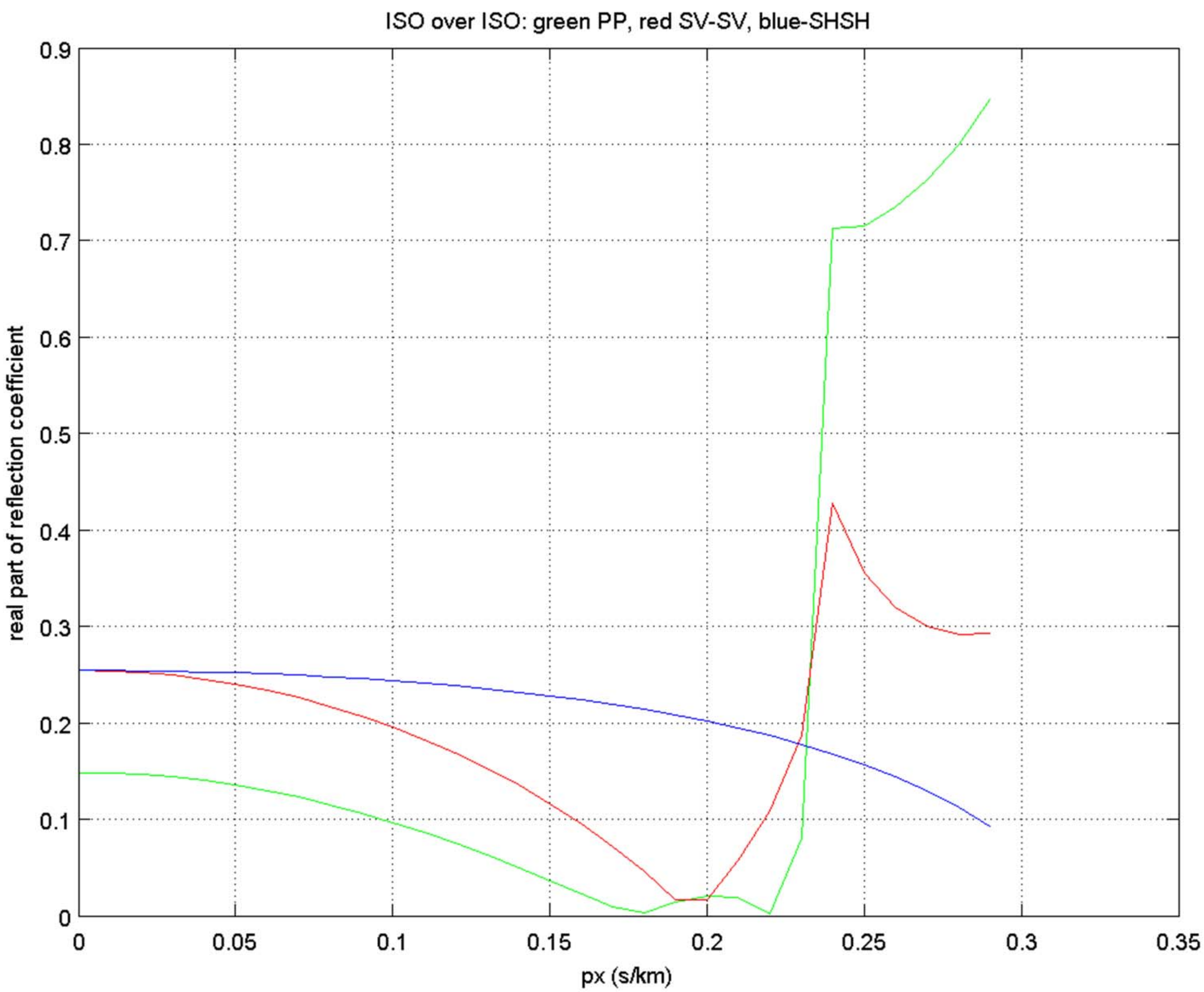


Elastic Wave Modeling : seismic modeling 1D part II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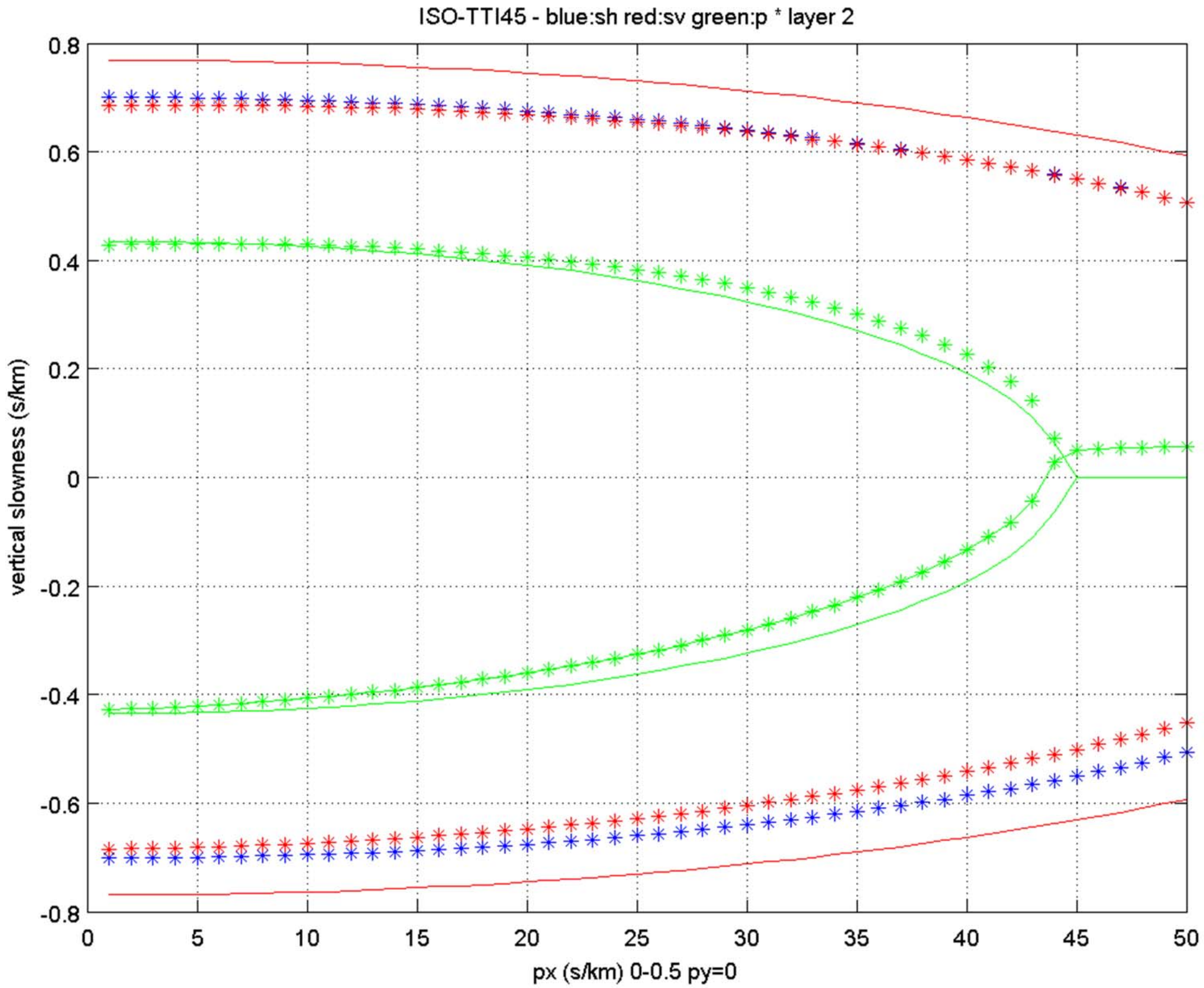
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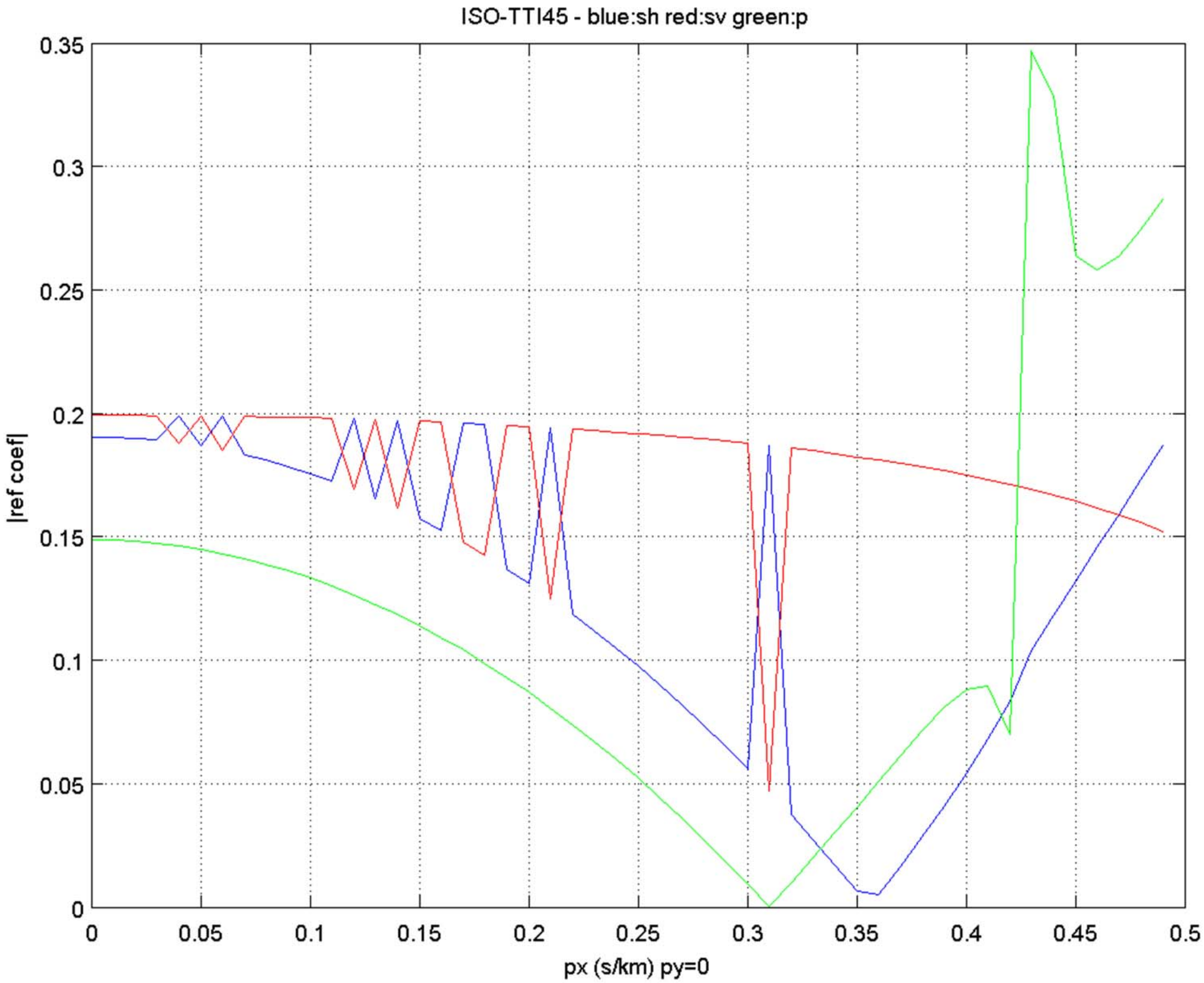
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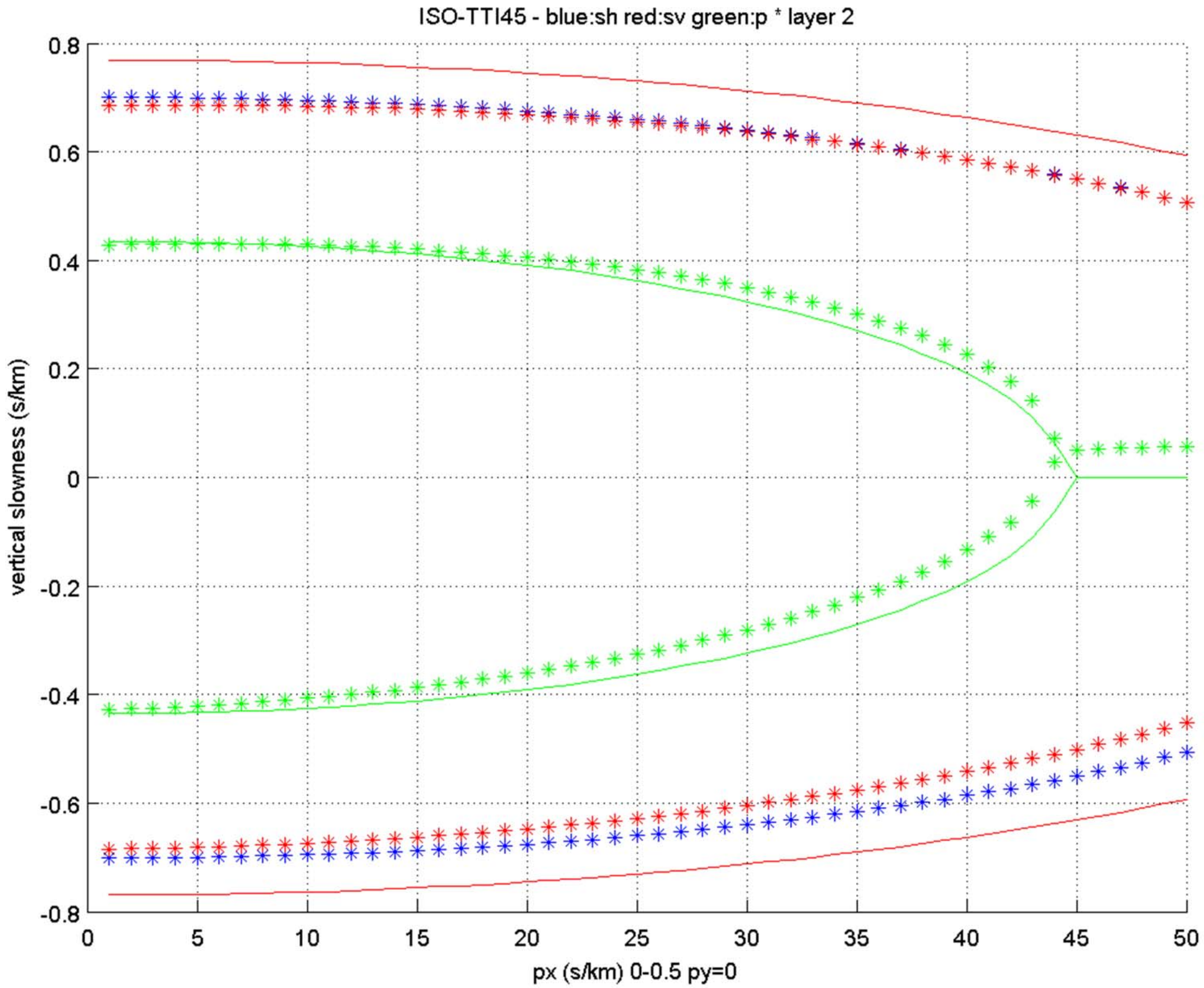

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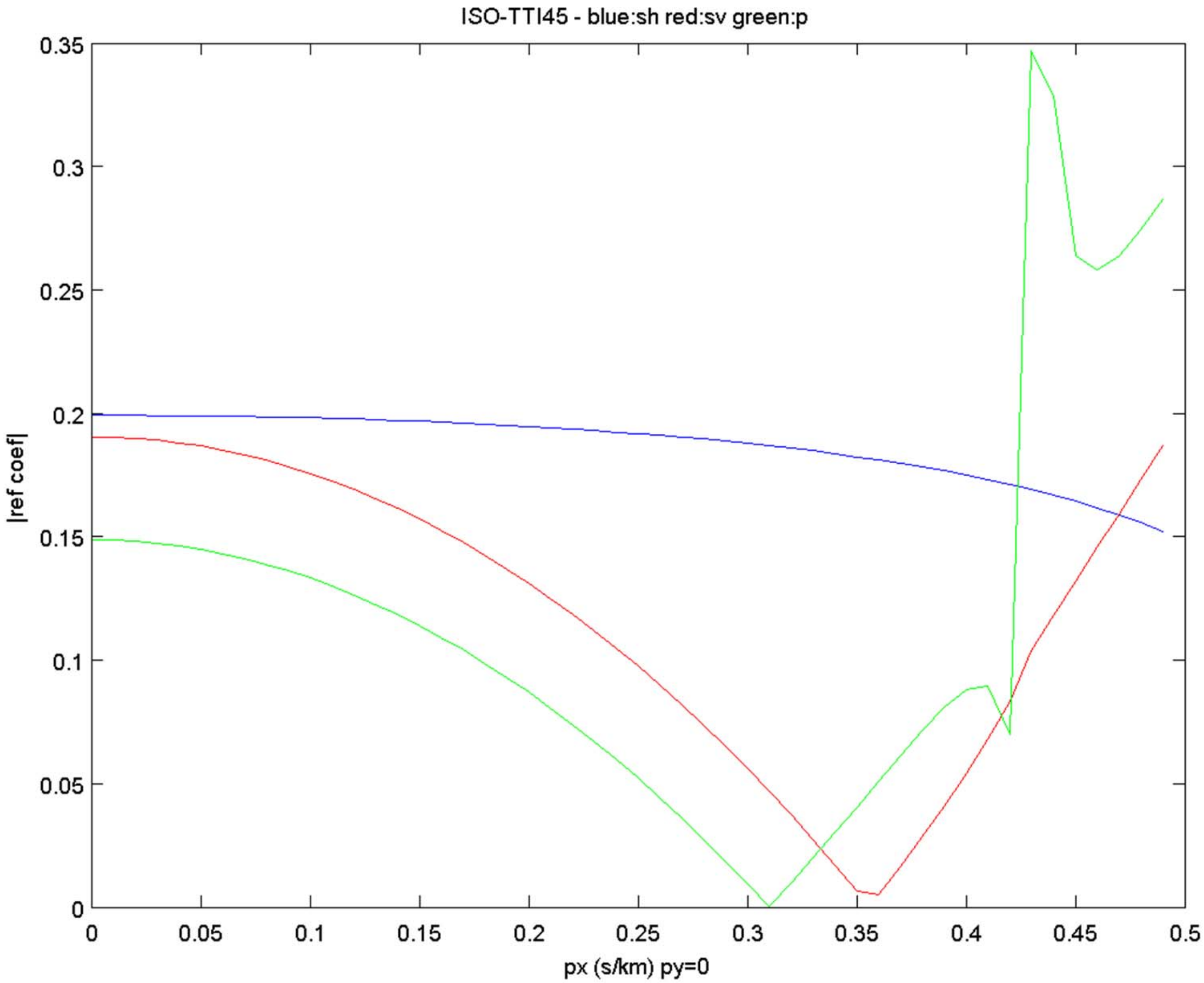
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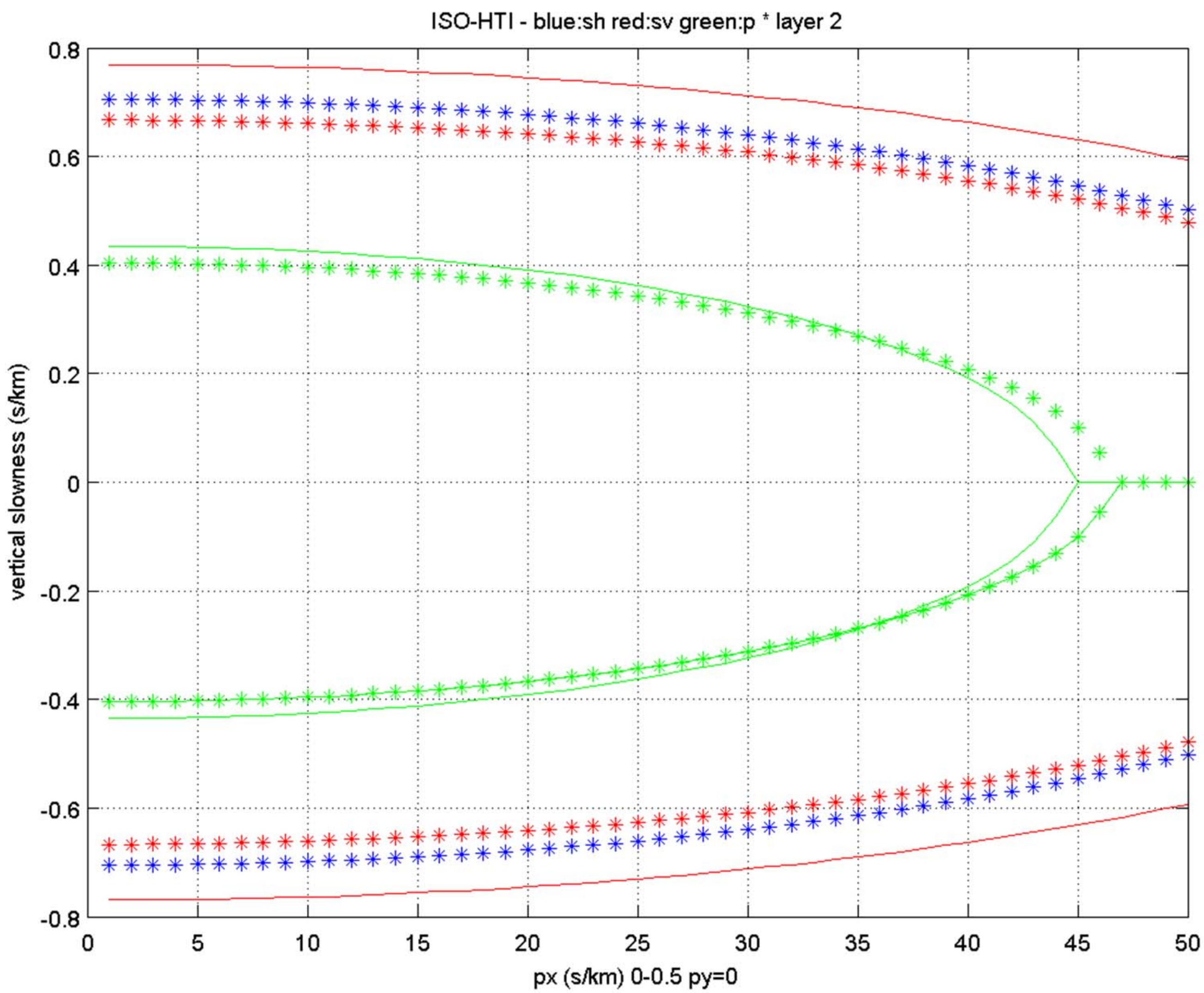
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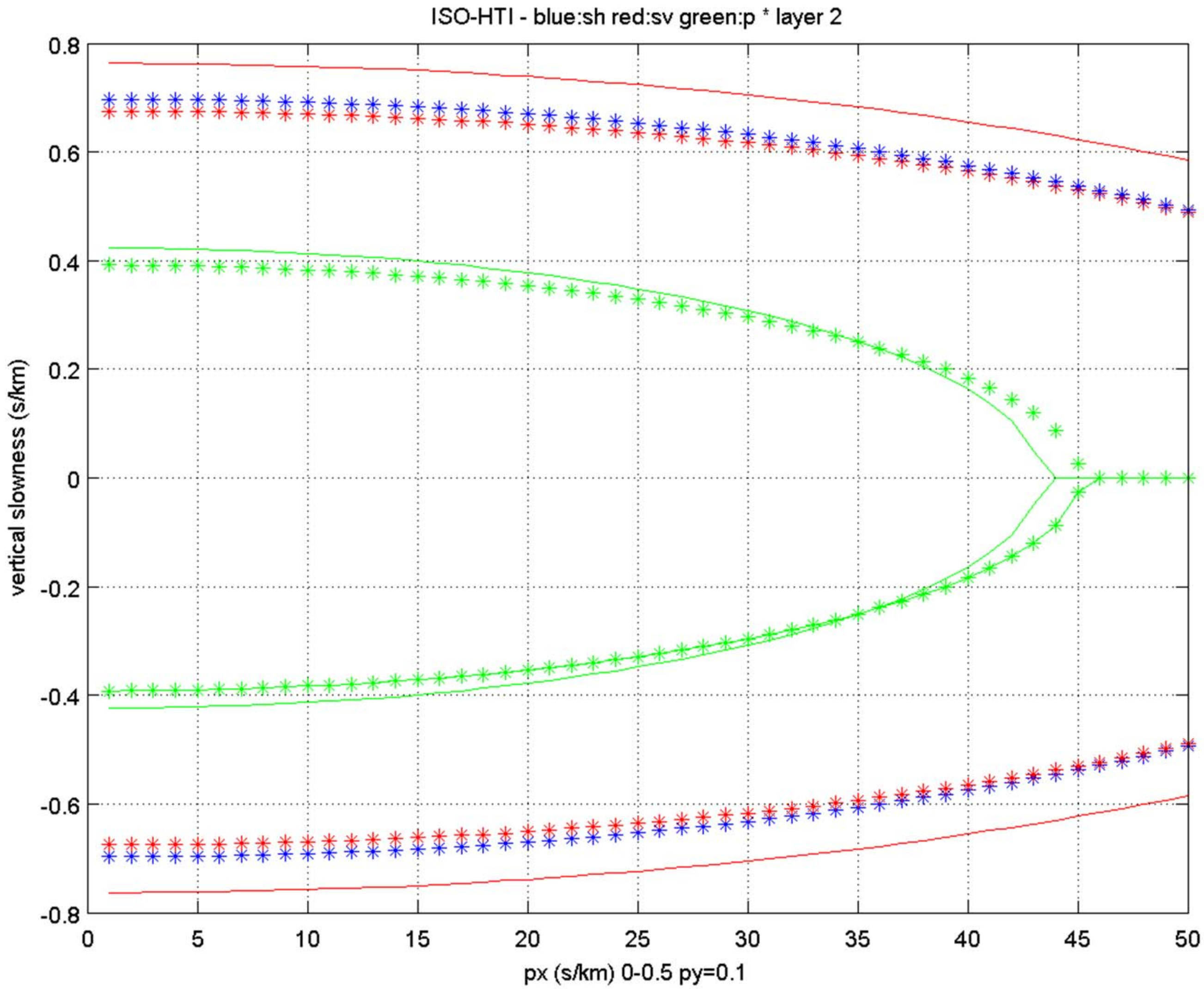
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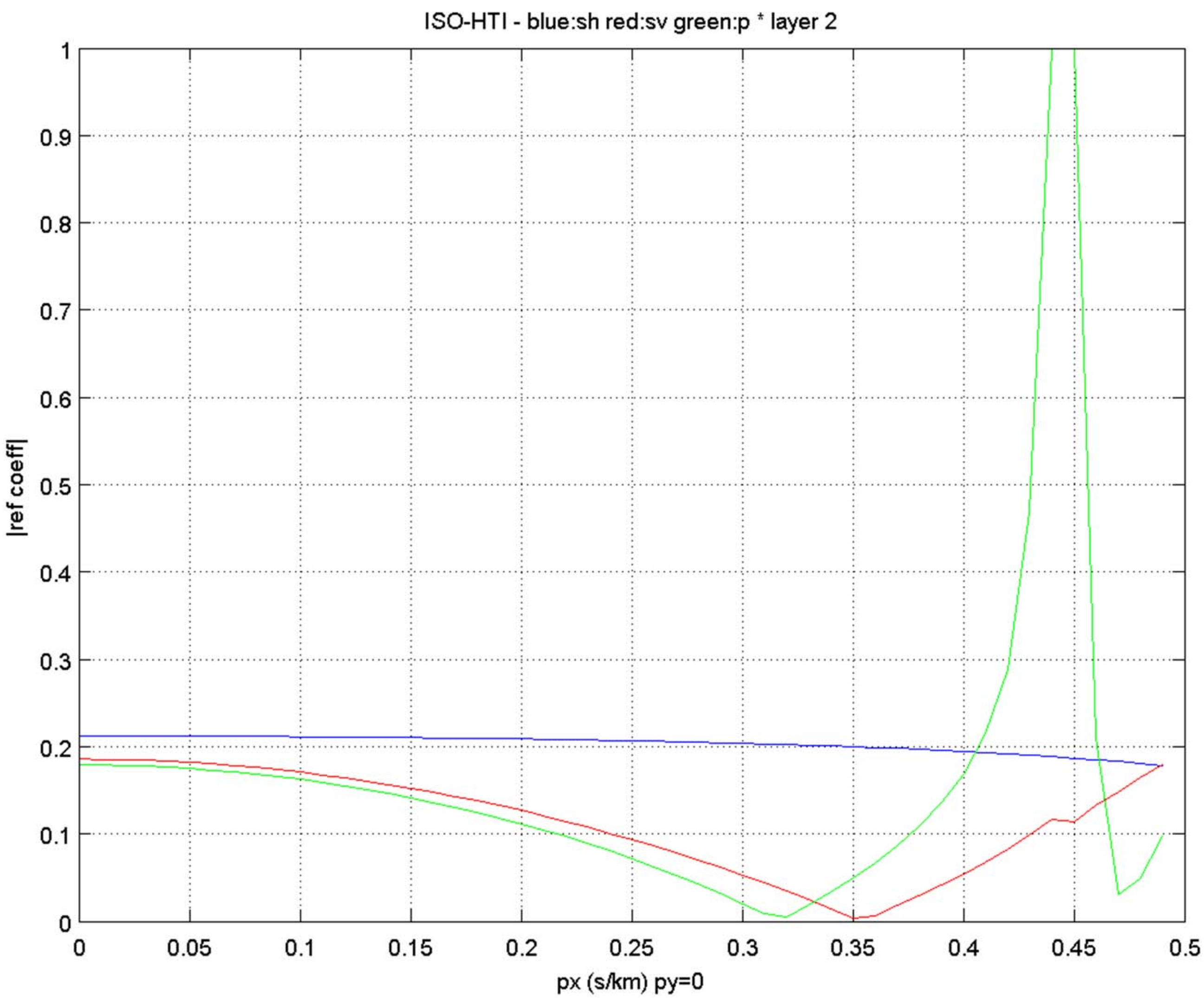
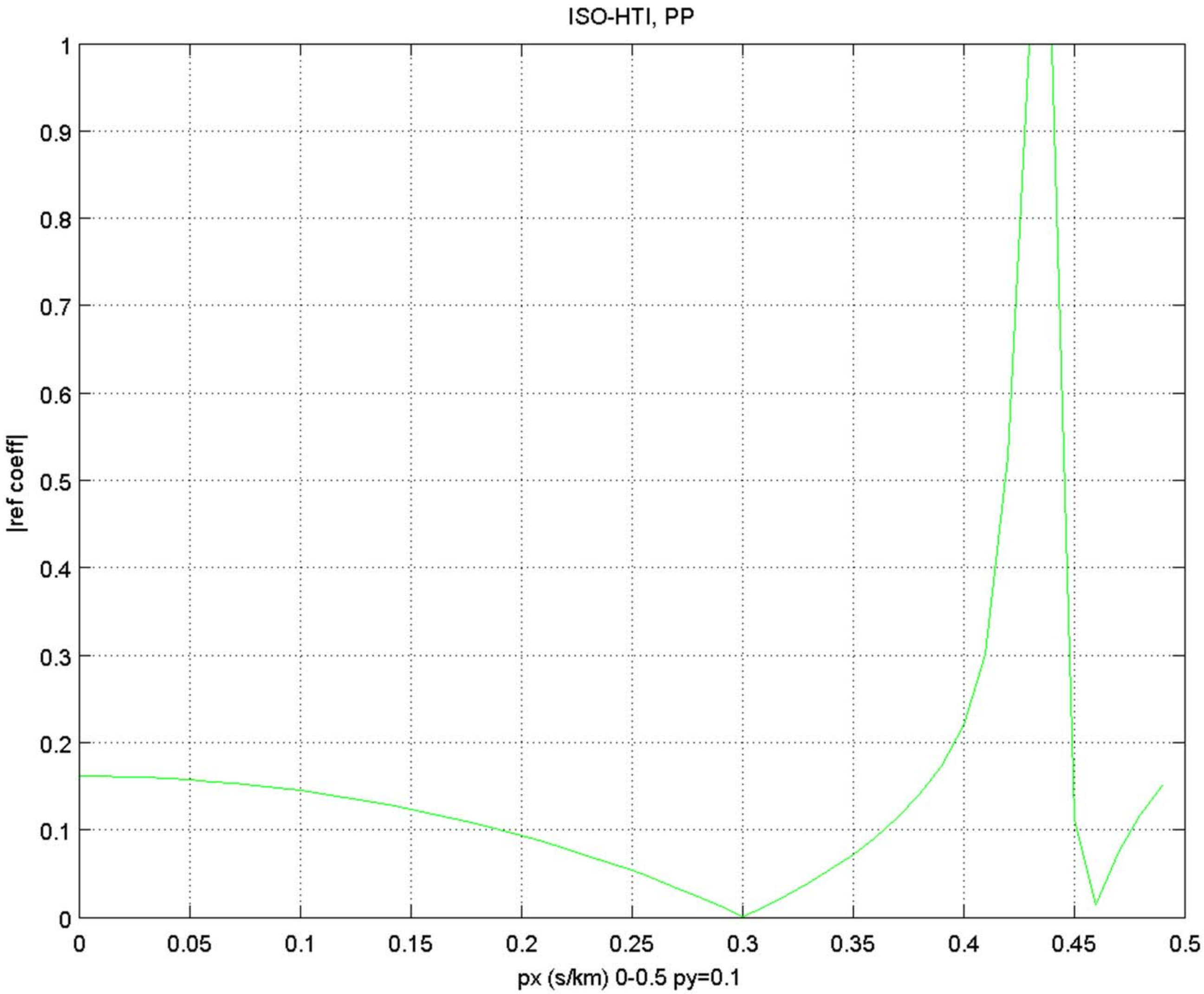


Elastic Wave Modeling : seismic modeling 1D part II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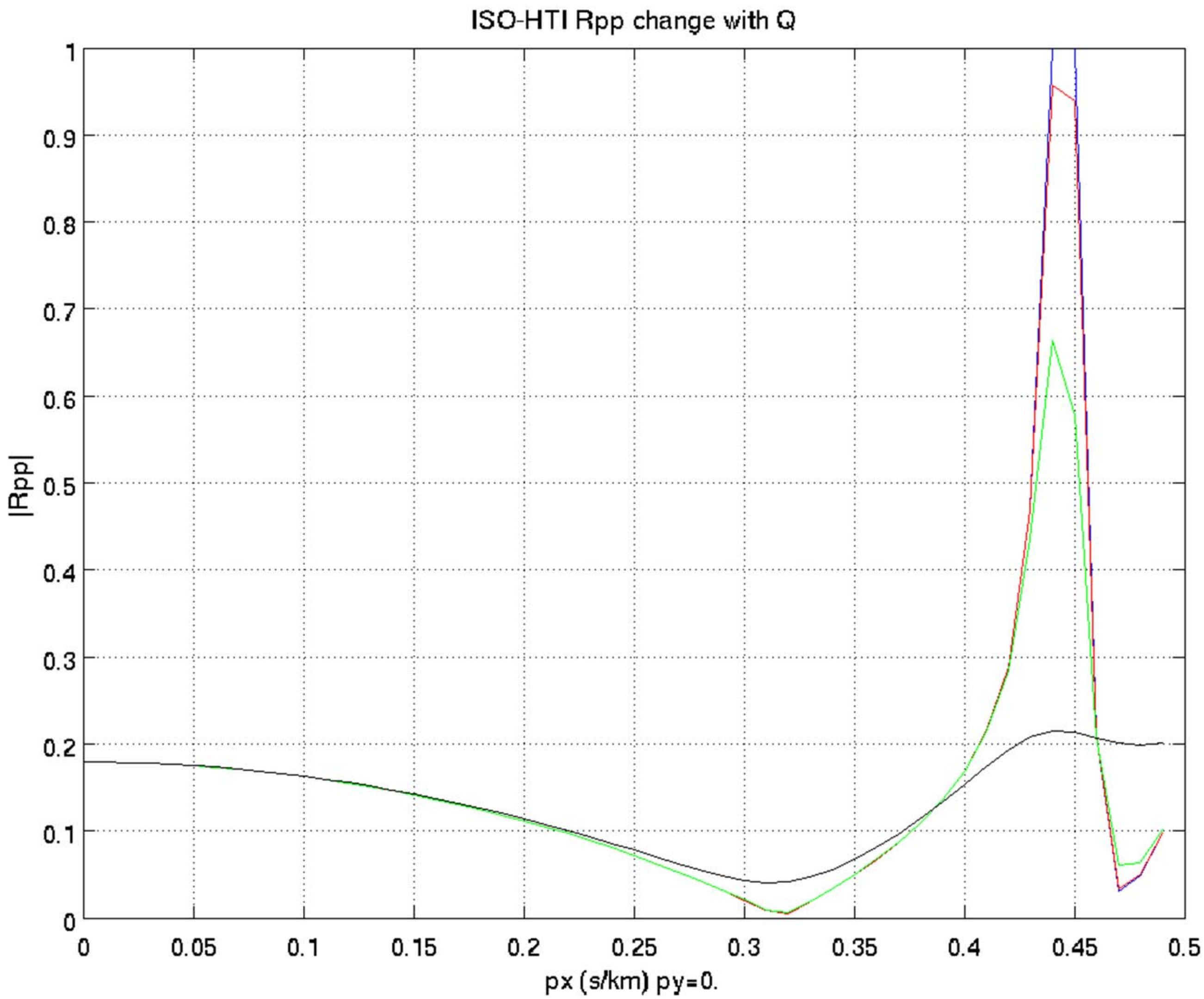

Elastic Wave Modeling : seismic modeling 1D part II

Summary

- Our work on Discontinuous Galerkin finite element method is ongoing
  - 3D
  - Include multiple fracture sets (pores, cracks)
- Interesting case studies with fractured porous media
- Frequency dependent AVO (spectrum versus offset)
- Frequency dependent shear wave splitting