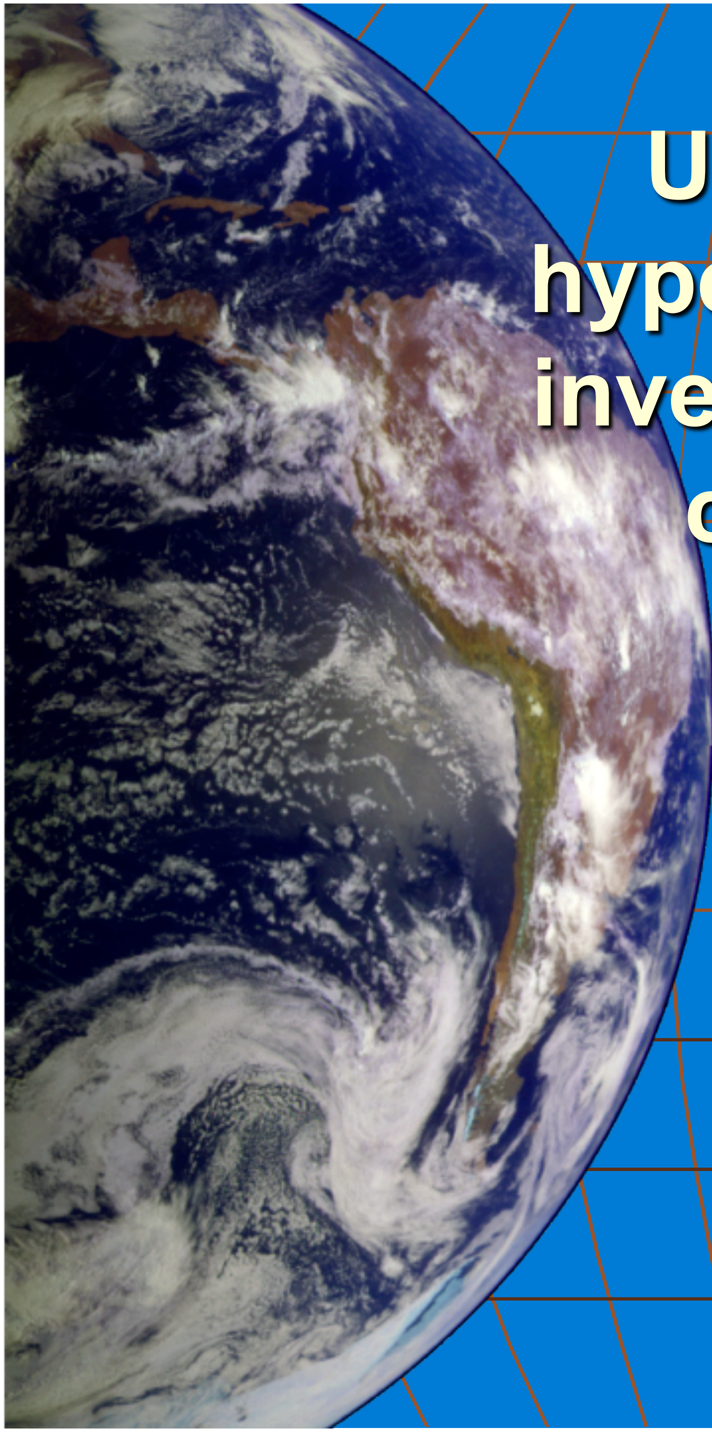

Use of priors and hyperpriors in seismic inversion for reservoir characterization

Mrinal K. Sen
Jackson School of Geosciences

EDGER 2/23/2010

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Goals

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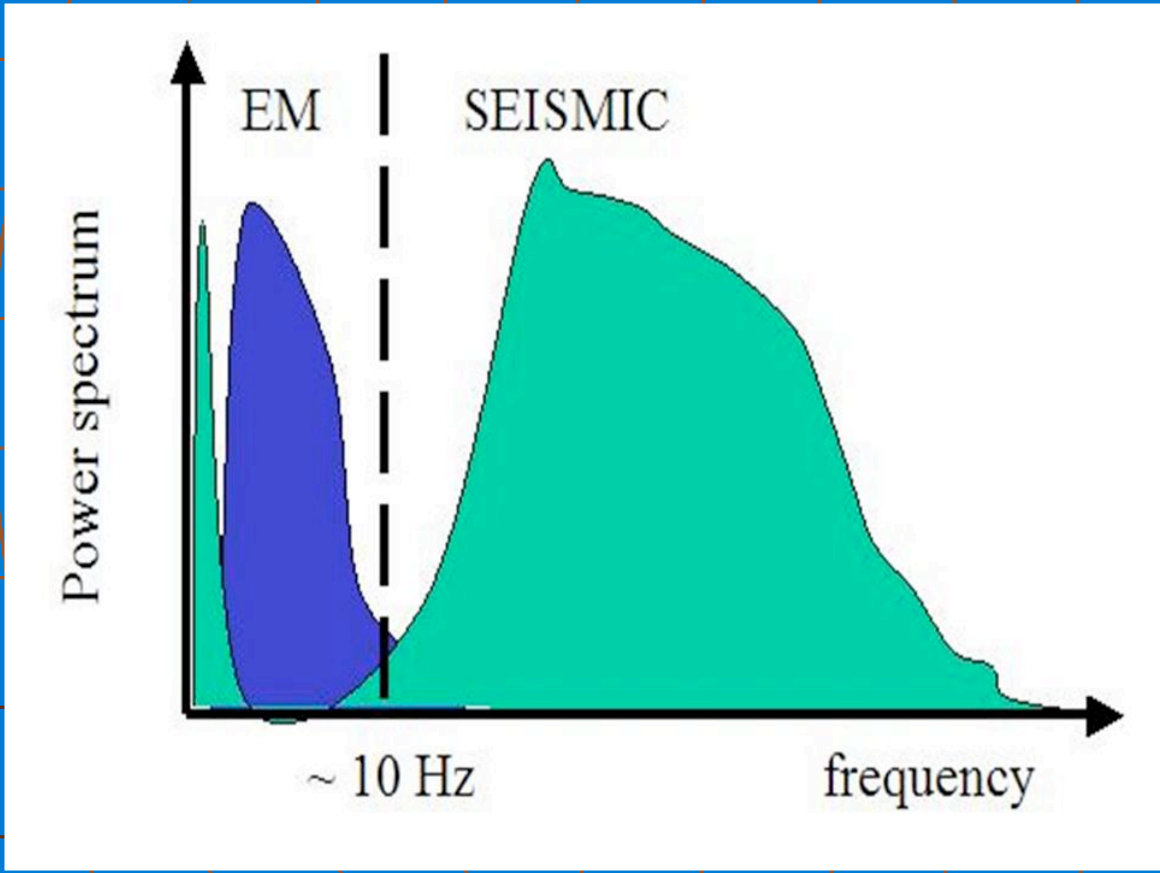
- An ultimate aim in reflection seismic exploration is to obtain a log of rock properties as a function of two-way vertical traveltime or depth → derive a pseudo-log

- Fill in the gaps between wells

Challenge

- Seismic data are almost always insufficient, inadequate and inconsistent
- Seismic data carry limited information on the subsurface
- How do we derive pseudo logs that are geologically meaningful and useful for reservoir characterization?

Band Limitation



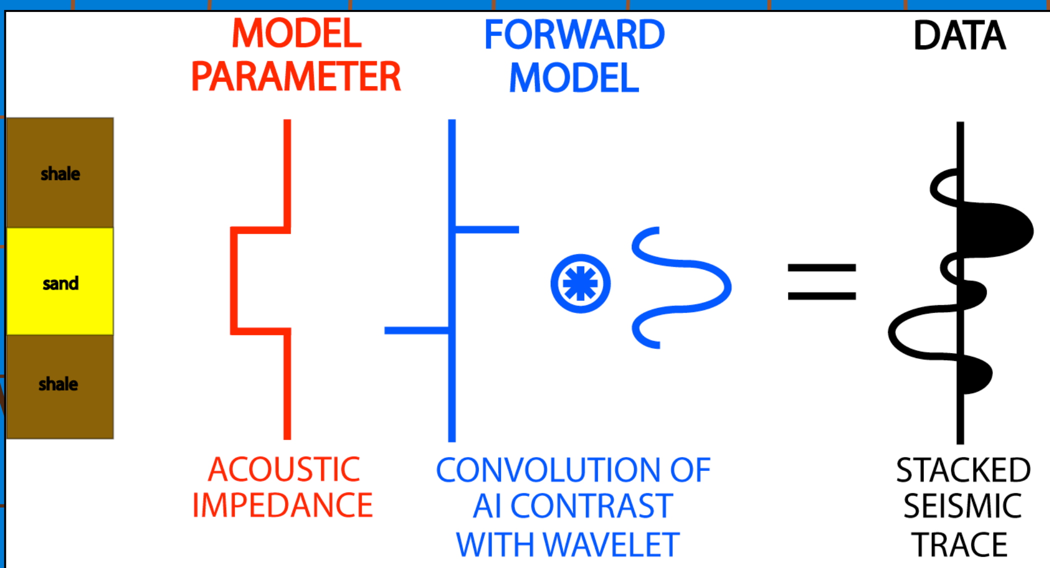
Frequency band limitation

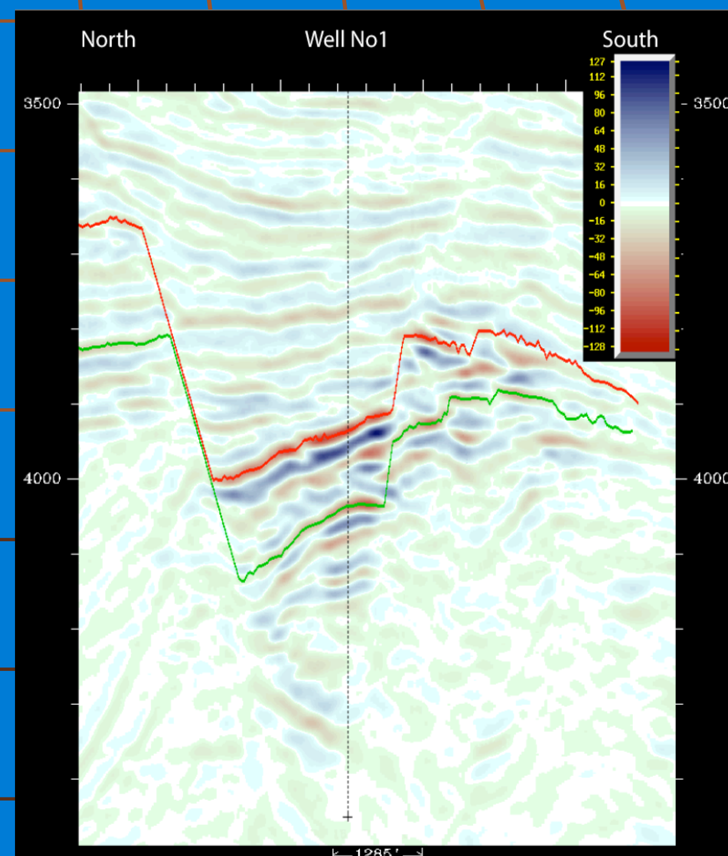
Offset limitation

Mukerji et al 2009

Example: Normal incidence seismograms

- Post-stack seismic trace





Forward Model

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- Impulse Response $$R(t) = (1/2)\, d[\ln A(t)]/dt.$$
- Fourier transform $$R(f) = (1/2) \int_0^{\infty} \left[\frac{d \ln A(t)}{dt} \right] \exp(i 2\pi f t)\, dt,$$

For Broadband, exact reconstruction is

$$\ln[A(t)/A_0] = 2 \int_0^t R(t')\, dt',$$

$$\ln[A(t)/A_0] = 2R(t) * H(t),$$

Ghosh 2000

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Normal Incidence

- Reality: band-limitation
- Low pass

$$\ln[A_b(t)/A_0] = 2R(t) * b(t) * H(t).$$

$$A_b(t) = A_0 \exp\{\ln[A(t)/A_0] * 2 f_H \operatorname{sinc} 2\pi f_H t\}$$

Ghosh 2000

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Normal Incidence data: implication of band limitation

- limited-frequency content results in violation of causality.
- A signal which is limited absolutely in frequency cannot be limited absolutely in duration so much that the signal cannot be identically zero in any interval of time.
- Thus, a reflection response beginning at *t = 0, by band limiting, would be smeared in time and, in a strict sense, would be nonzero for most of the negative values of t .*
- *This evidently runs counter to the principle of causality, in that an observation exists in time much before it actually originates.*

Ghosh 2000

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Normal Incidence data: implication of band limitation

- *This is an inevitable artifact of band limiting. Therefore, although it is enough to invert the full-band impulse response from some reference time t = 0, the band-limited impulse response must be inverted starting from t = −∞.*
- *Fortunately, for practical purposes and a typical seismic band, the amount of anticausality is small—often a few milliseconds.*
- *Apart from the anticausal feature, band limiting also distorts time relationship in the reverse way: It delays information, making it spill over into the future, owing to the same fundamental result of signal processing.*

Ghosh 2000

Normal Incidence data: implication of band limitation

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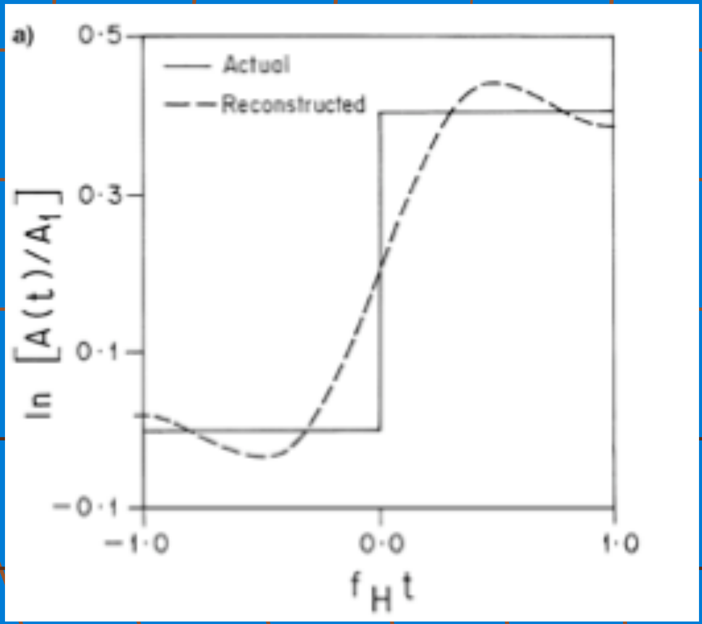
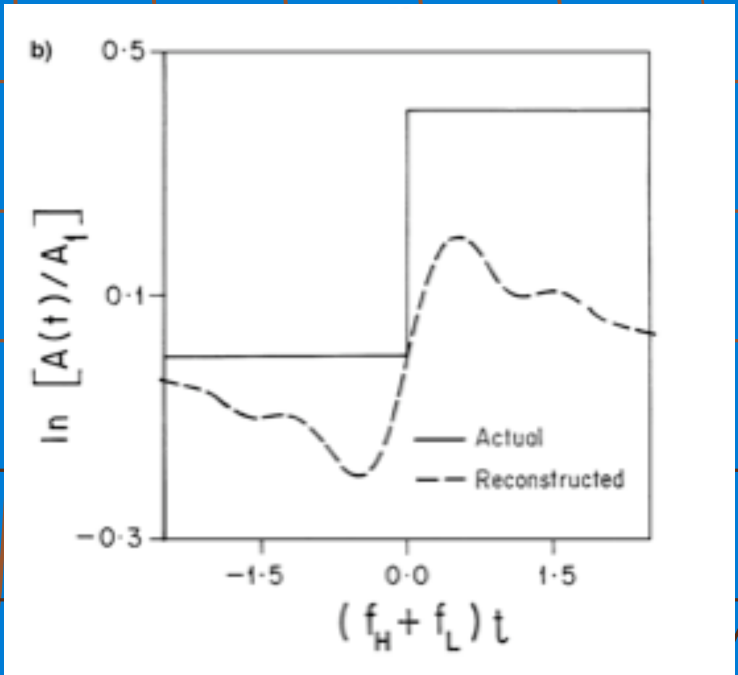
- Low-pass

Band-pass





Ghosh 2000

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Normal Incidence data: implication of band limitation

- Shift in the estimated absolute value of acoustic impedance
- Blocky or layered nature of the well log is lost
- High frequency variations are missing

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Post-Stack Inversion Workflow
Deterministic Inversion

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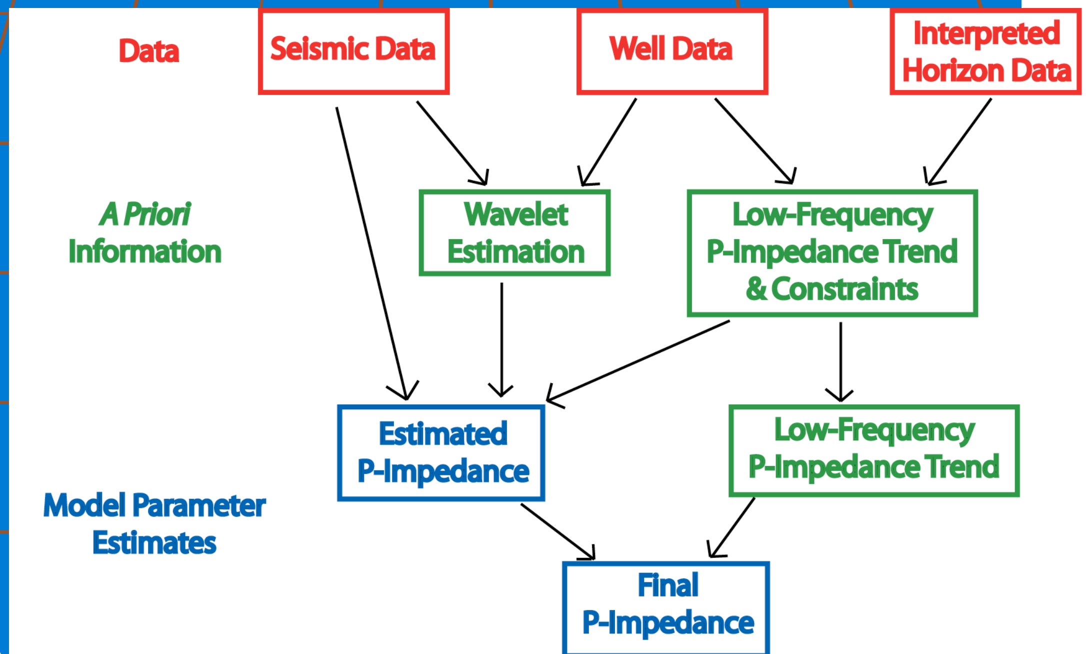

Minimize

$$E(\mathbf{m}) = \lambda \left\| \mathbf{d}_{\text{obs}} - g(\mathbf{r}(\mathbf{m})) \right\|_2 + \left\| \mathbf{r}(\mathbf{m}) \right\|_{0.9} + \alpha \left\| \Delta Zp \right\|_1$$

subject to

$$Zp_{\min;i} < Zp_i < Zp_{\max;i}$$

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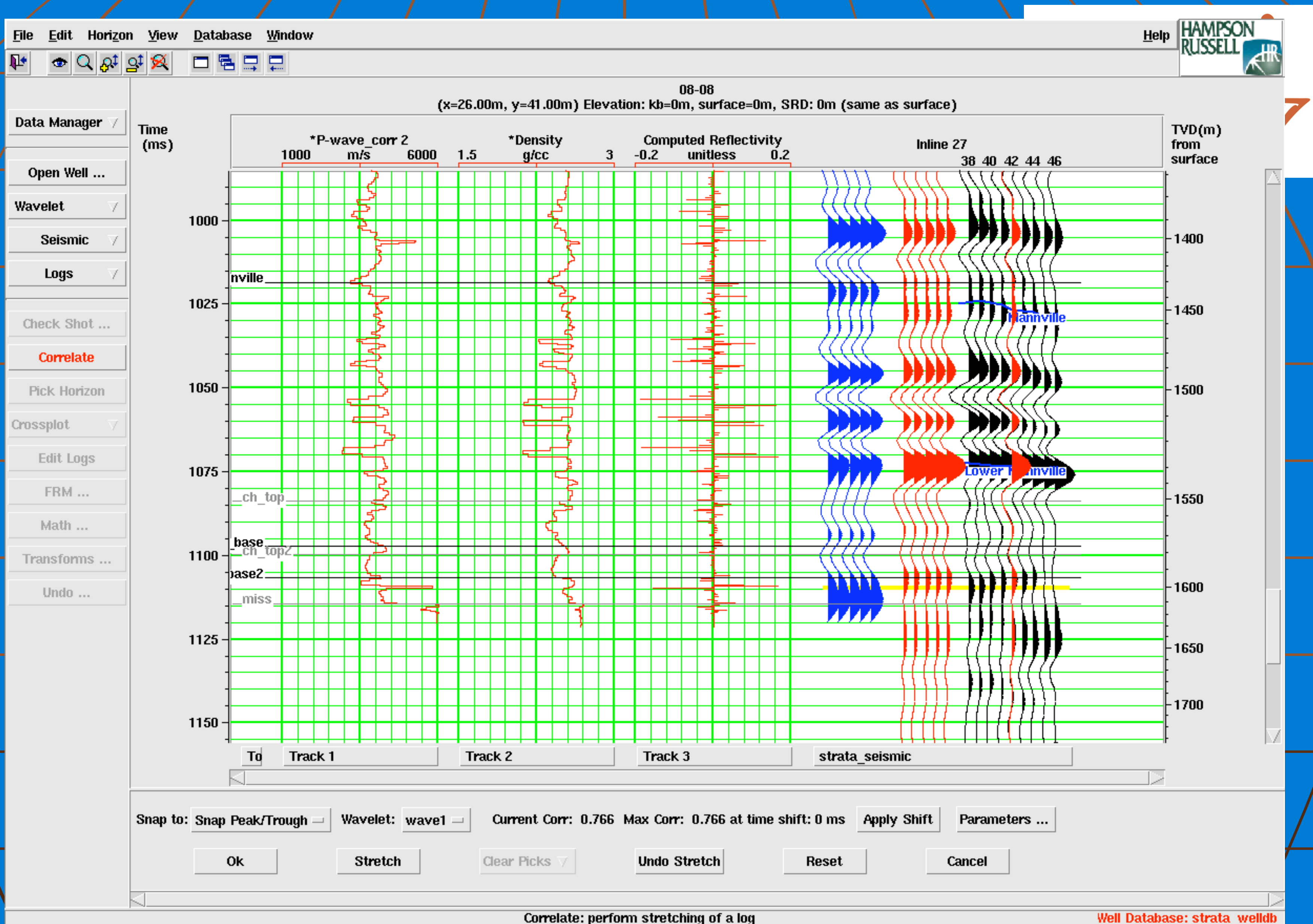

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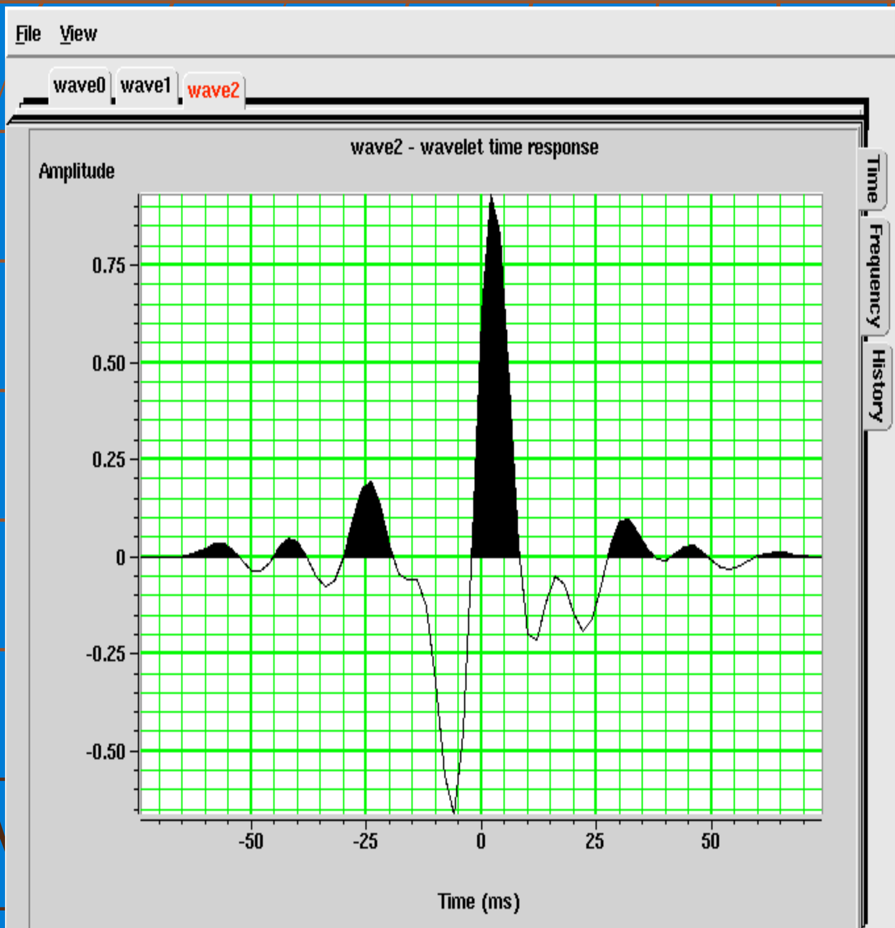

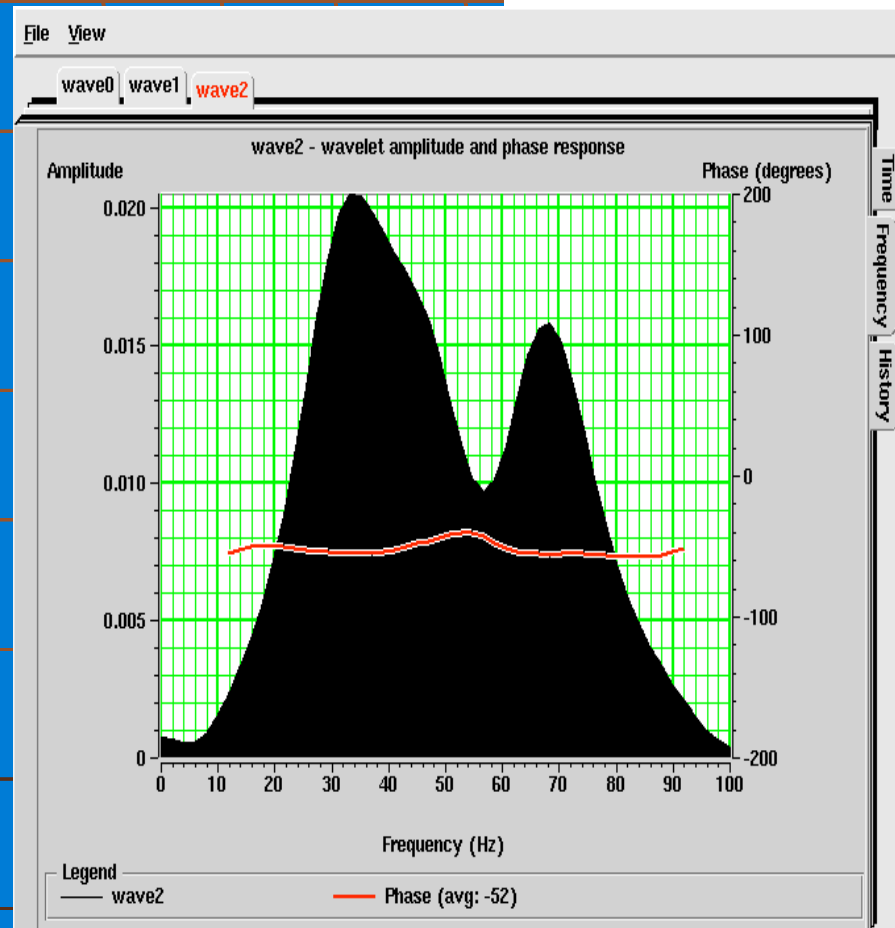

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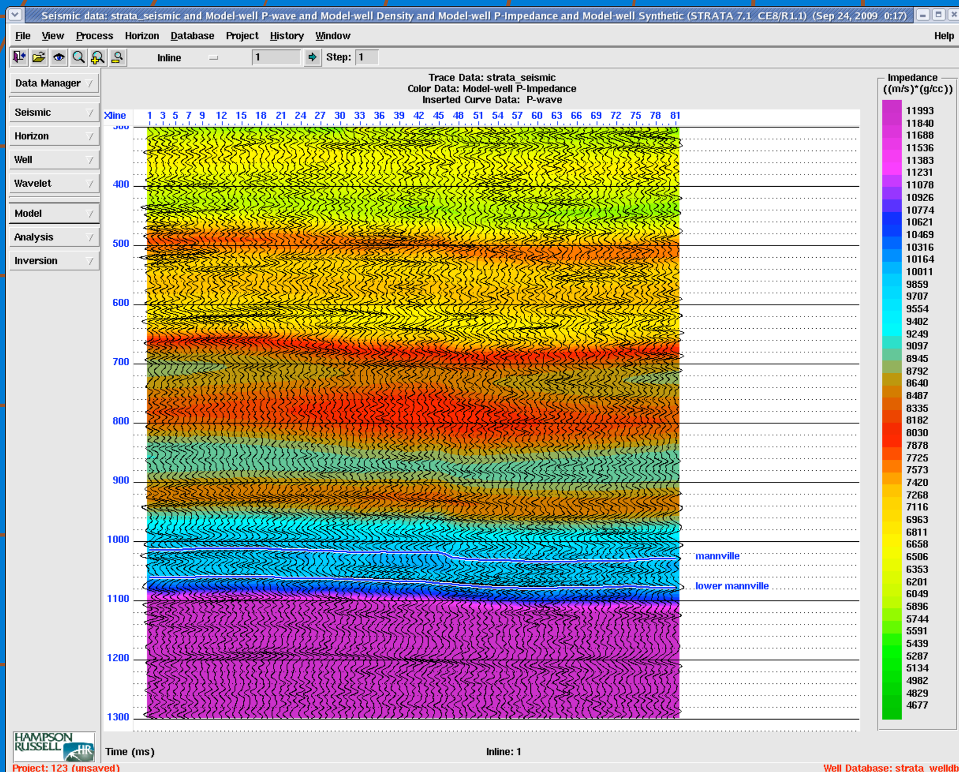
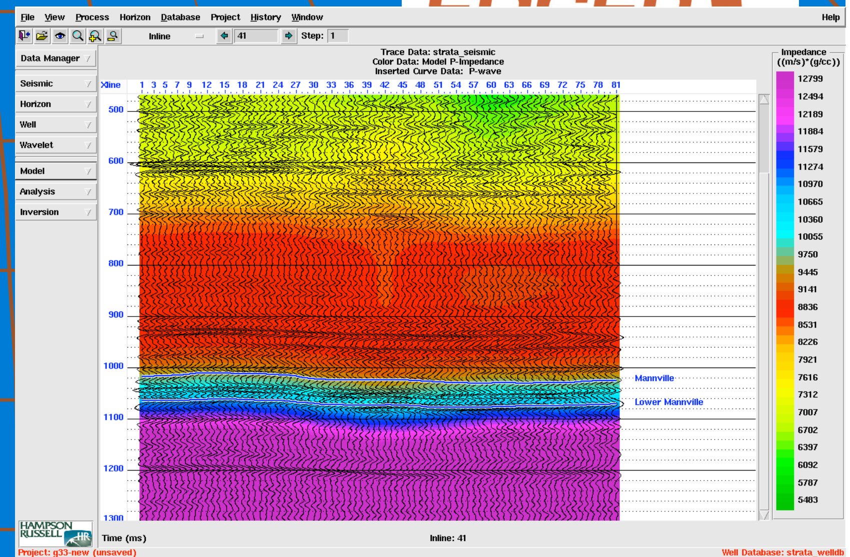
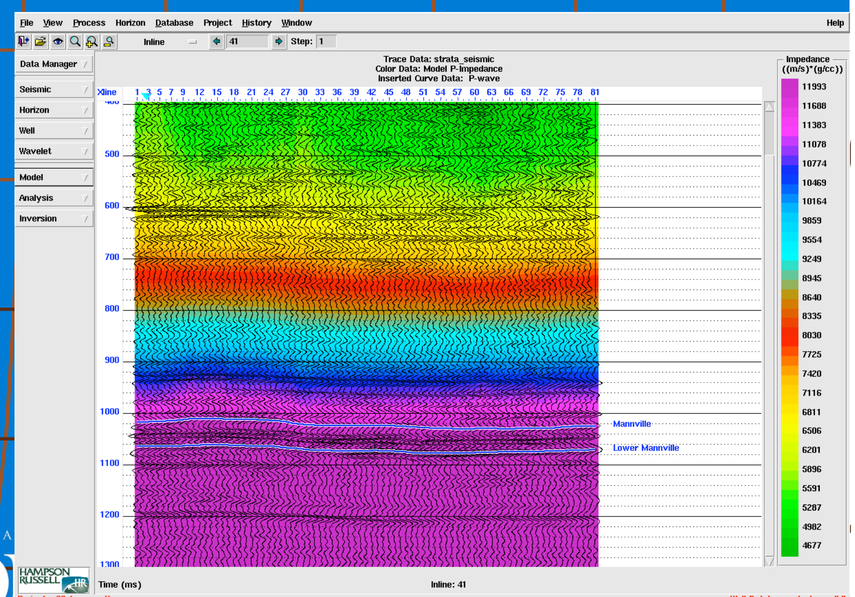
Initial model for different high cut frequency



10/15Hz

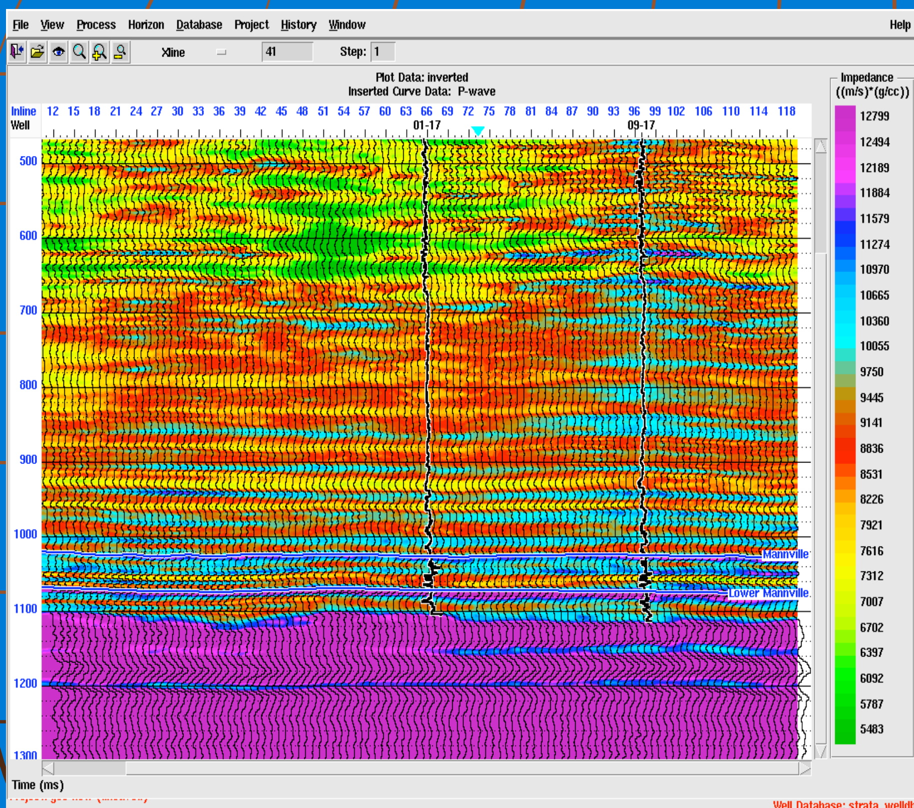


3/5Hz

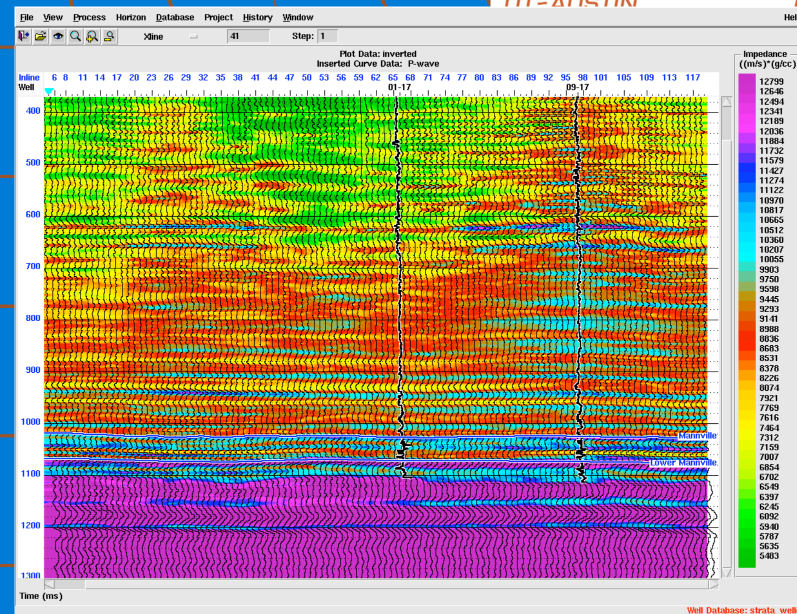


0.5/1Hz

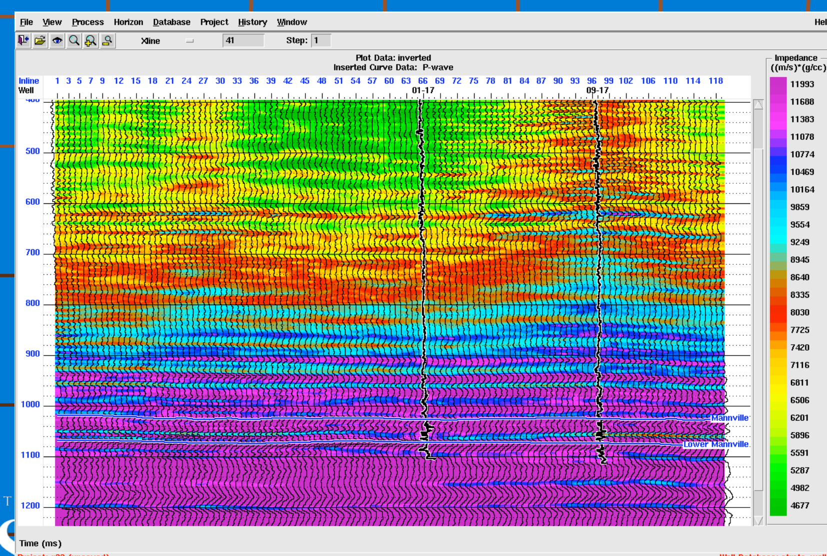
Inverse model



10/15Hz



3/5Hz



0.5/1Hz

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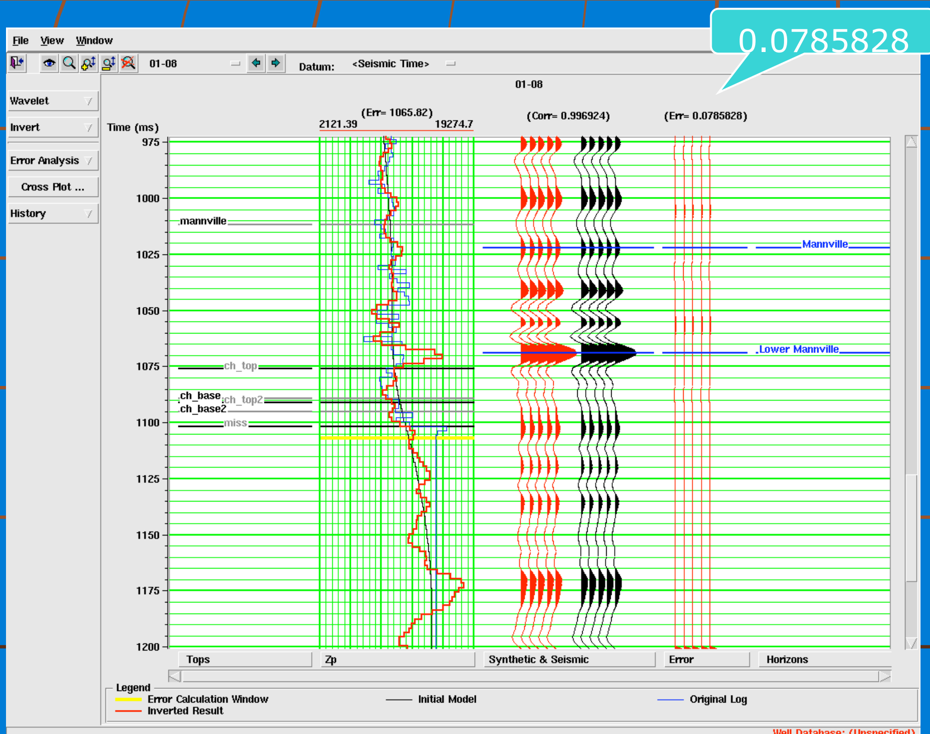
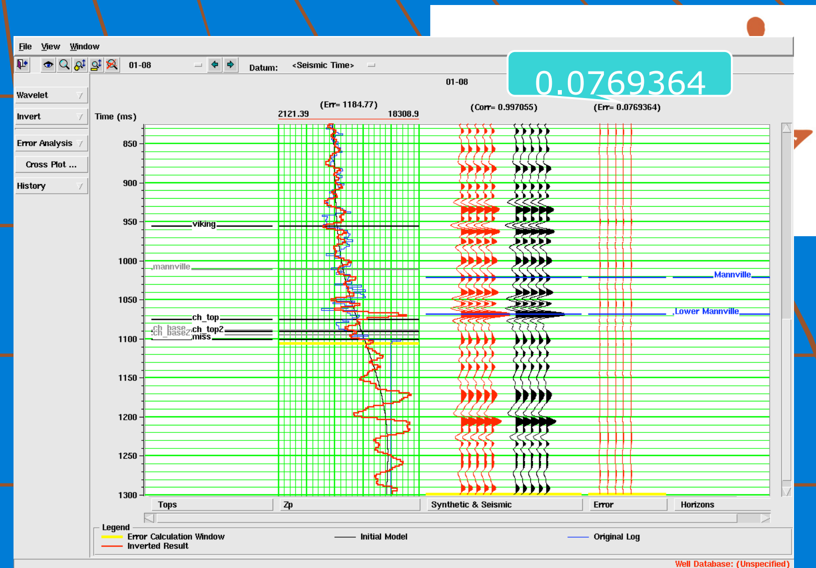
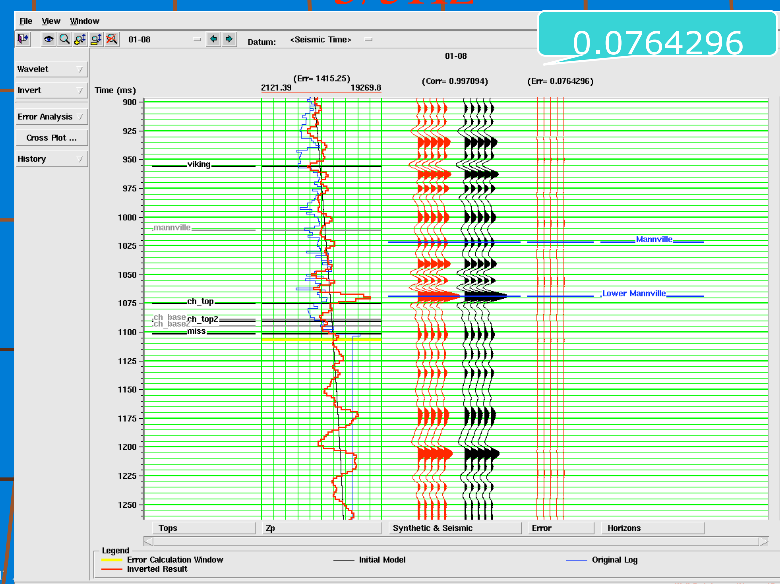
Post-stack analysis



10/15Hz



3/5Hz



0.5/1Hz

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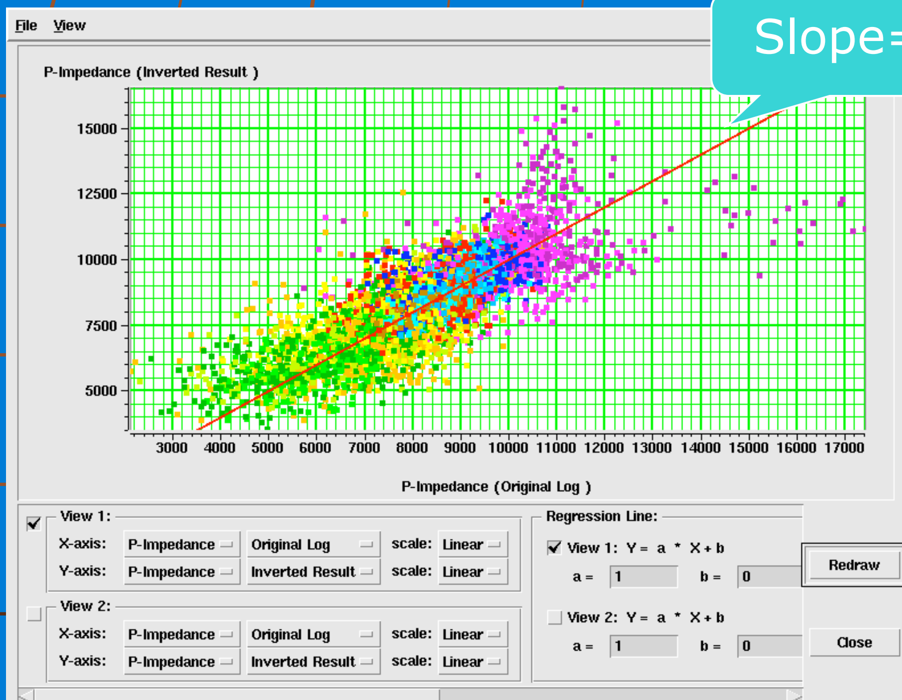
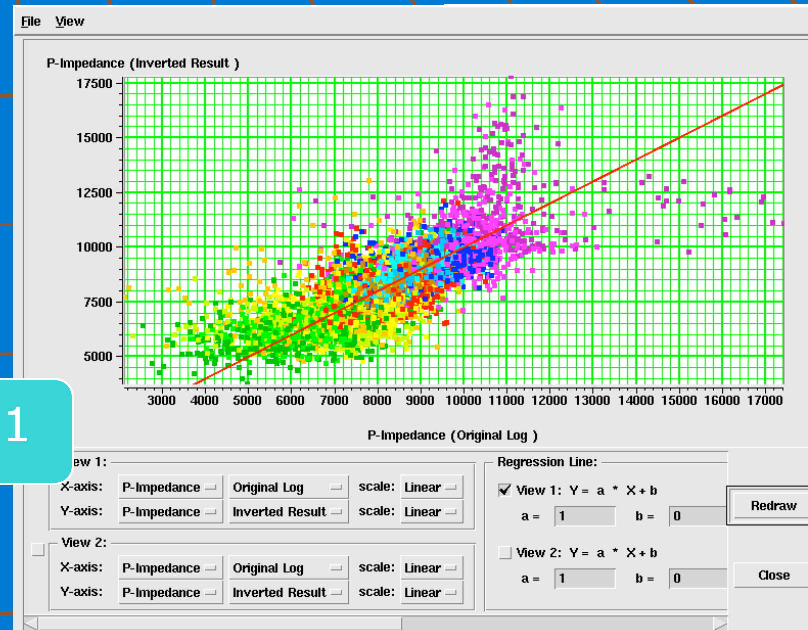
Cross-plot

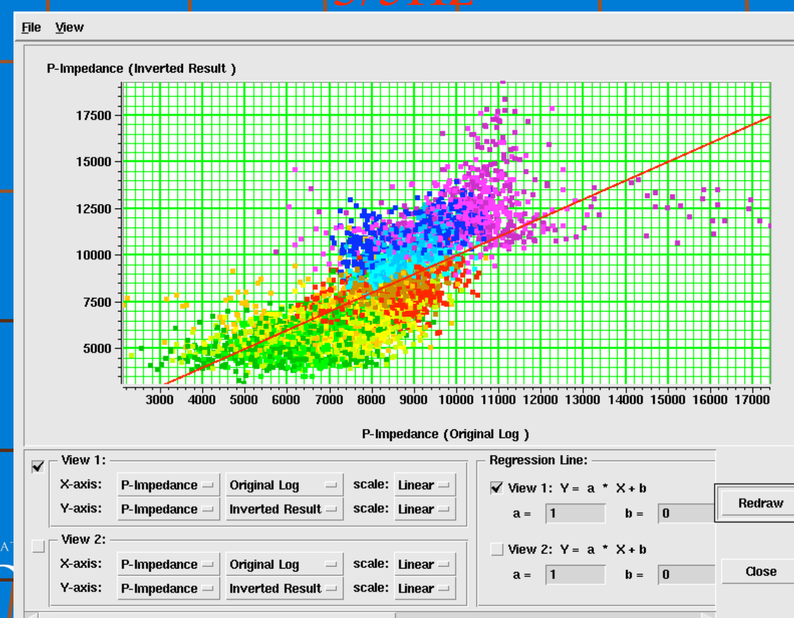


Slope=1

10/15Hz



3/5Hz



0.5/1Hz

How to incorporate spikiness in reflectivity- blockyness in impedance?

- Use Non-smooth Constraints - regularization
  - Lp norm
  - Total Variation
  - **Gaussian Hyper-prior**
- Choose appropriate basis
  - Wavelets
  - Curvelets

Routh and Sen 2008

Formulation of Inverse Problem

- Minimize model objective function

$$\phi_m = R(m)$$

A priori Information

- Subject to fitting the data

$$\phi_d = \left\| W_d \left(d^{OBS} - F(m) \right) \right\|^2$$

Information from Physics

- Global objective function

$$\phi(m) = \phi_d + \beta \; R(m)$$

Regularization

Choices for Non-Smooth Model Objectives

- Lp Norm

$$R(m) = \left(\int |m|^p \; dv \right)^{\frac{1}{p}}$$

Spiky Solution

e.g.: reflectivity

- Total Variation

$$R(m) = \int \sqrt{|\nabla m|^2 + \alpha} \; dv$$

Blocky Solution

e.g.: Interval velocity, Tomography

- Compactness Constraint

$$R(m) = \int \frac{m^2}{m^2 + \alpha} \, dv$$

Localized Changes

e.g.: Time-Lapse

Inversion in Wavelet Domain using L1 norm

- Consider Linear system $d^{obs} = Gm + \varepsilon$
  - Expand the model in Wavelet Basis:

$$m(r) = \sum_k C_{j_0,k}\phi_{j_0,k}(r) + \sum_{j=j_0}^{\infty}\sum_k d_{j,k}\psi_{j,k}(r)$$

$$d^{obs} = GW^T Wm + \varepsilon$$

$$d^{obs} = \tilde{G}\tilde{m} + \varepsilon$$

- Solve the inverse problem:

$$\min \phi = \beta \left\| D\tilde{m} \right\|_1 + \left\| W_d \left(d^{obs} - \tilde{G}\tilde{m} \right) \right\|^2$$

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Routh et. al , 2007

Bayesian Inverse Problem

- Bayes

$$P\left(m \middle| d^{obs}\right) \propto P\left(d^{obs} \middle| m\right) P(m)$$

Likelihood Information

$$\exp\left(-\left\|W_d(d - Gm)\right\|^2\right)$$

Prior Information

$$\exp\left(-\left\|W_m m\right\|^2\right)$$

$$\exp\left(-\left\|W_m m\right\|_1\right)$$

$$\max \quad \exp\left(-\left[\left\|W_d(d - Gm)\right\|^2 + \beta\left\|W_m m\right\|^2\right]\right)$$

Bayesian Hyper-Prior Formulation

- Hyper-prior formulation

$$P(m,\theta|d^{obs}) \propto P(d^{obs}|m)P(m,\theta)$$

$$P(m,\theta|d^{obs}) \propto P(d^{obs}|m)P(m|\theta)P(\theta)$$

Prior Distribution Parameters : θ

P(θ) : Prior PDF for θ

Bayesian Hyper-Prior Formulation

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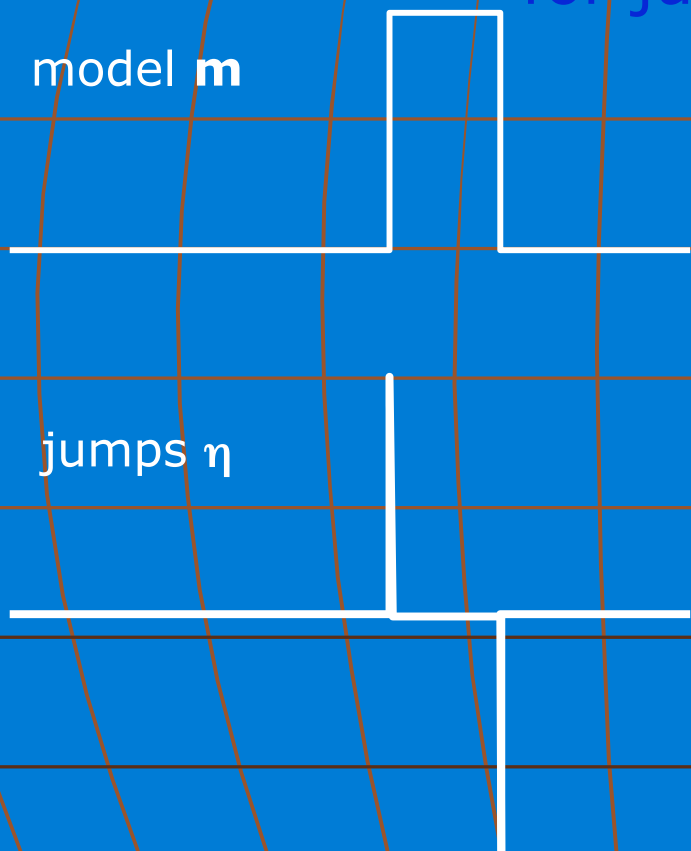
- Main Idea: Instead of solving for model solve for jumps



$$\eta_k = m_k - m_{k-1}$$

$$\eta = Bm$$

$$m = B^{-1}\eta$$

$$d^{obs} = Gm + \varepsilon$$

$$d^{obs} = \left(GB^{-1}\right)\eta + \varepsilon$$

Bayesian Hyper-Prior Formulation

- Choosing the Probability Distribution

$$P(\eta,\theta|d^{obs}) \propto P(d^{obs}|\eta)P(\eta|\theta)P(\theta)$$

Gaussian Distribution

Gaussian Distribution

Cauchy Distribution

Promotes Sparsity

Caveletti and Somerascelo, 2007

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Bayesian Hyper-Prior Formulation

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- Choosing the Probability Distribution

$$P(\eta,\theta | d^{obs}) \propto P(d^{obs} | \eta) P(\eta | \theta) P(\theta)$$

$$\exp\left[-\left\| W_d \left(d - GB^{-1}\eta \right) \right\|^2 - \left\| \sqrt{\theta}\eta \right\|^2 - \gamma \left\| \theta \right\|_1 + \frac{1}{2} \sum_{k=1}^{M} \log \theta_k \right]$$

Inversion with Bayesian Hyper-Prior Formulation

Break the optimization problem in two parts:

- Solve for η with fixed θ

$$\min_{\eta} \quad \left\| W_d \left(d - G B^{-1} \eta \right) \right\|^2 + \frac{1}{2} \left\| \sqrt{\theta}\, \eta \right\|^2$$

- Solve for θ with fixed η (close form solution)

$$\theta = diag\left(\frac{1}{\eta + \gamma} \right)$$

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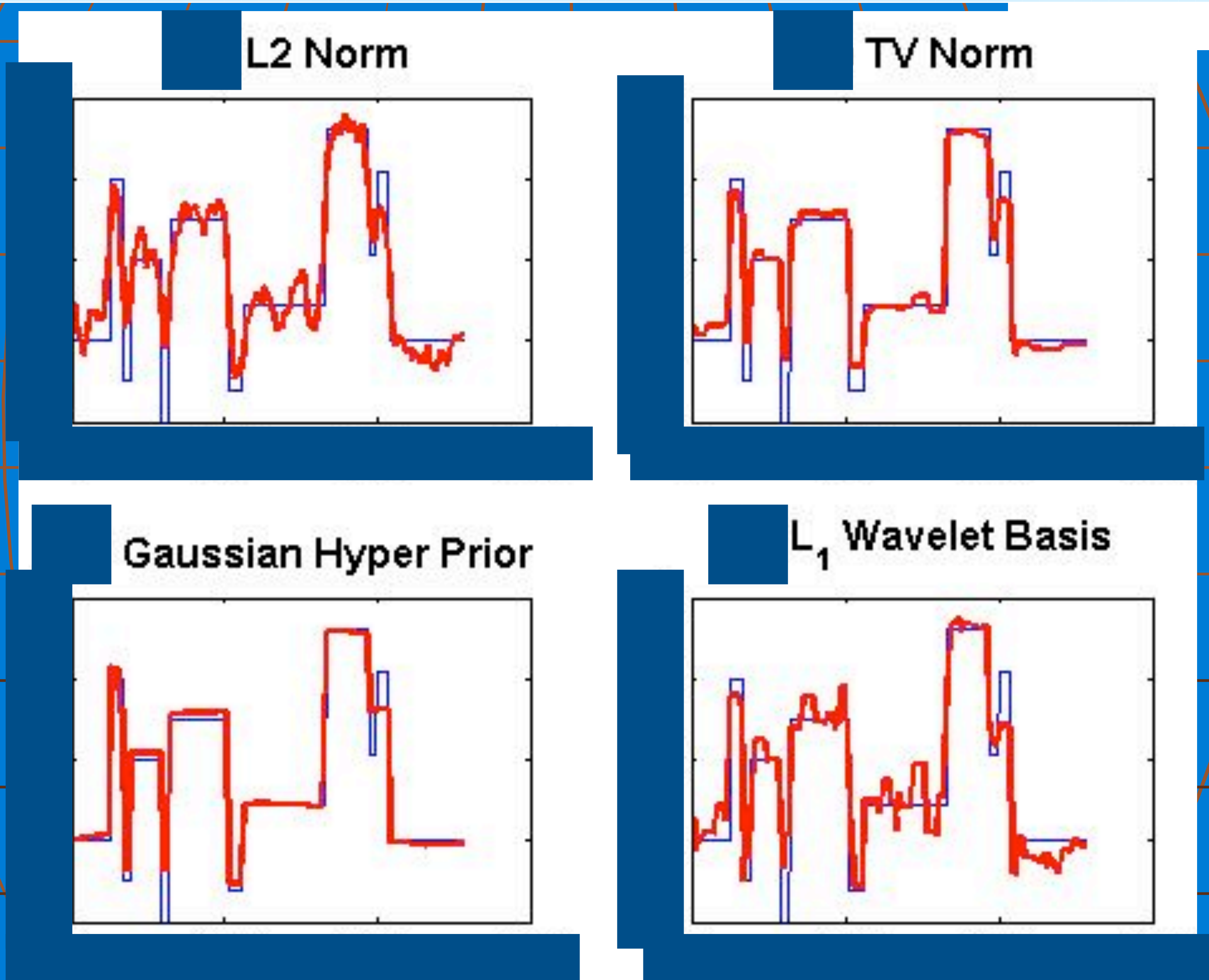
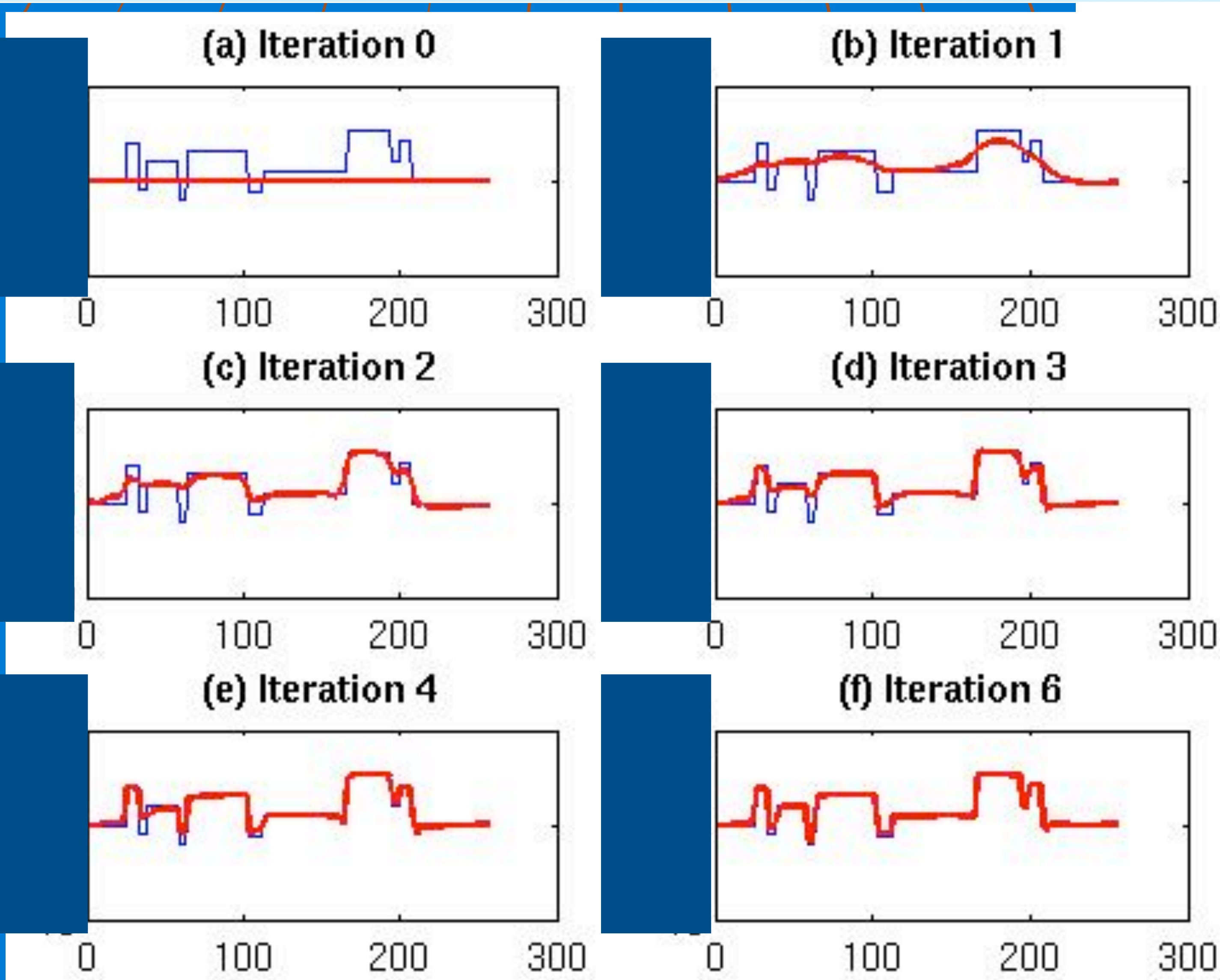
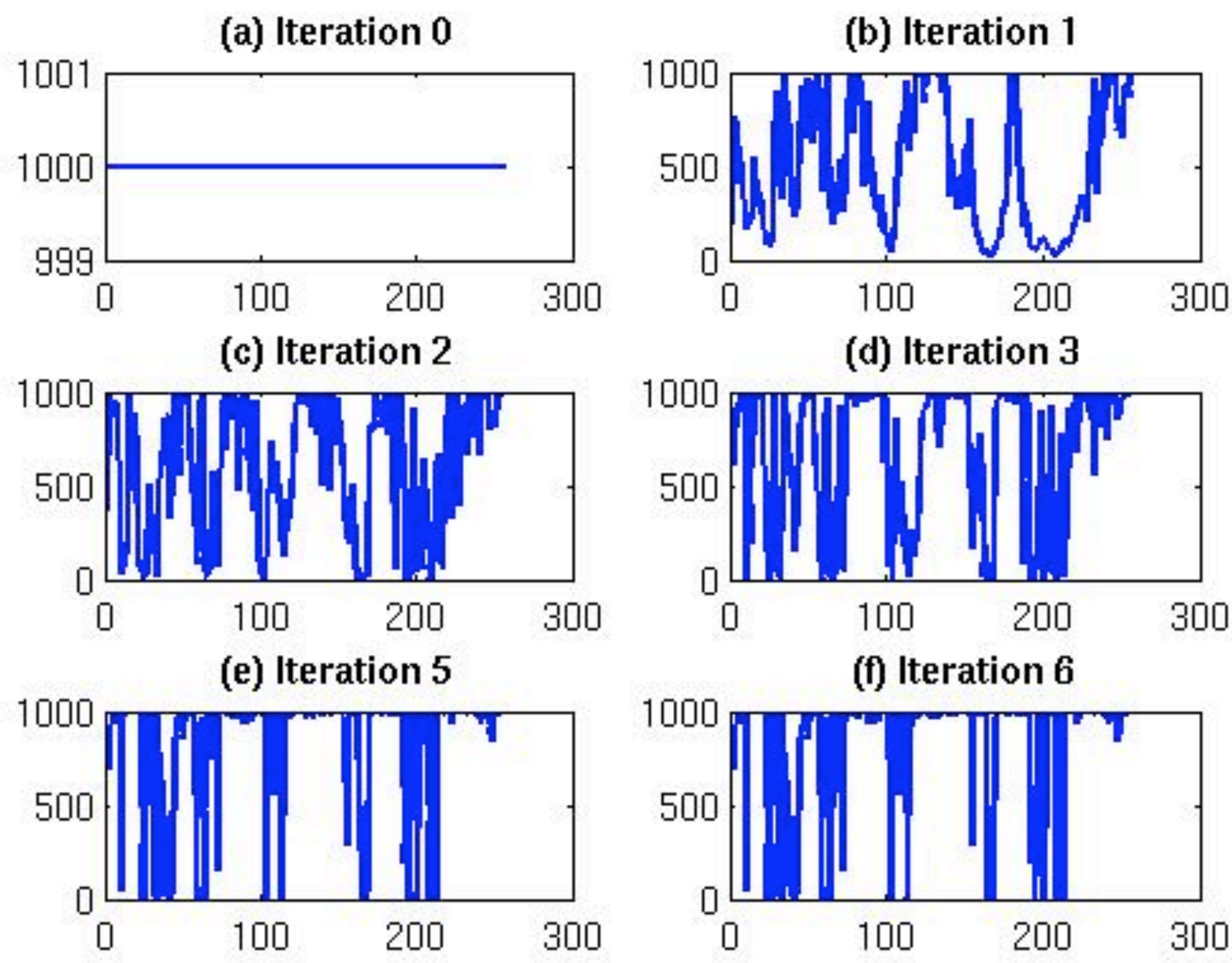
Deconvolution Problem (S/N = 3)



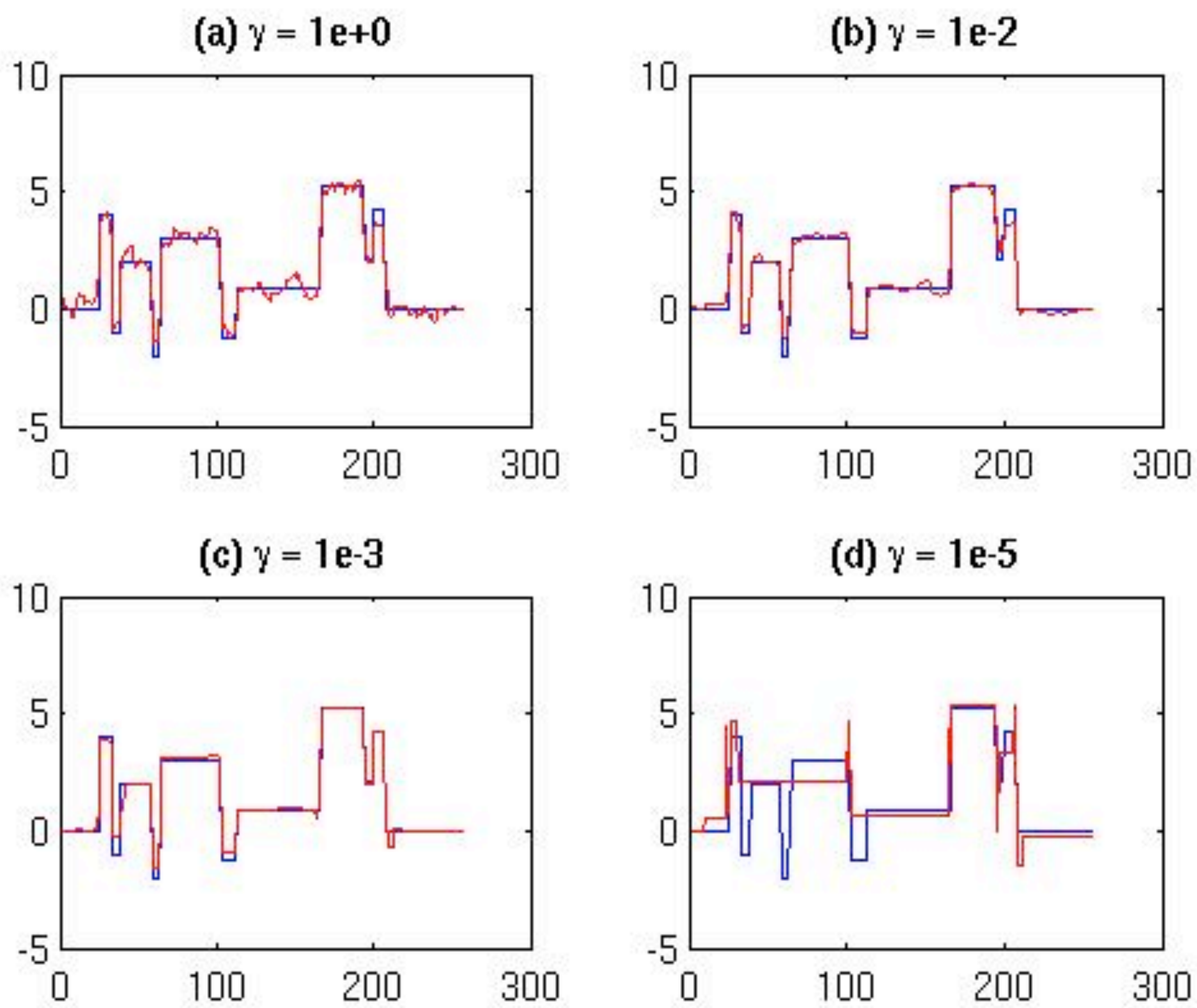
Iterative Model Construction



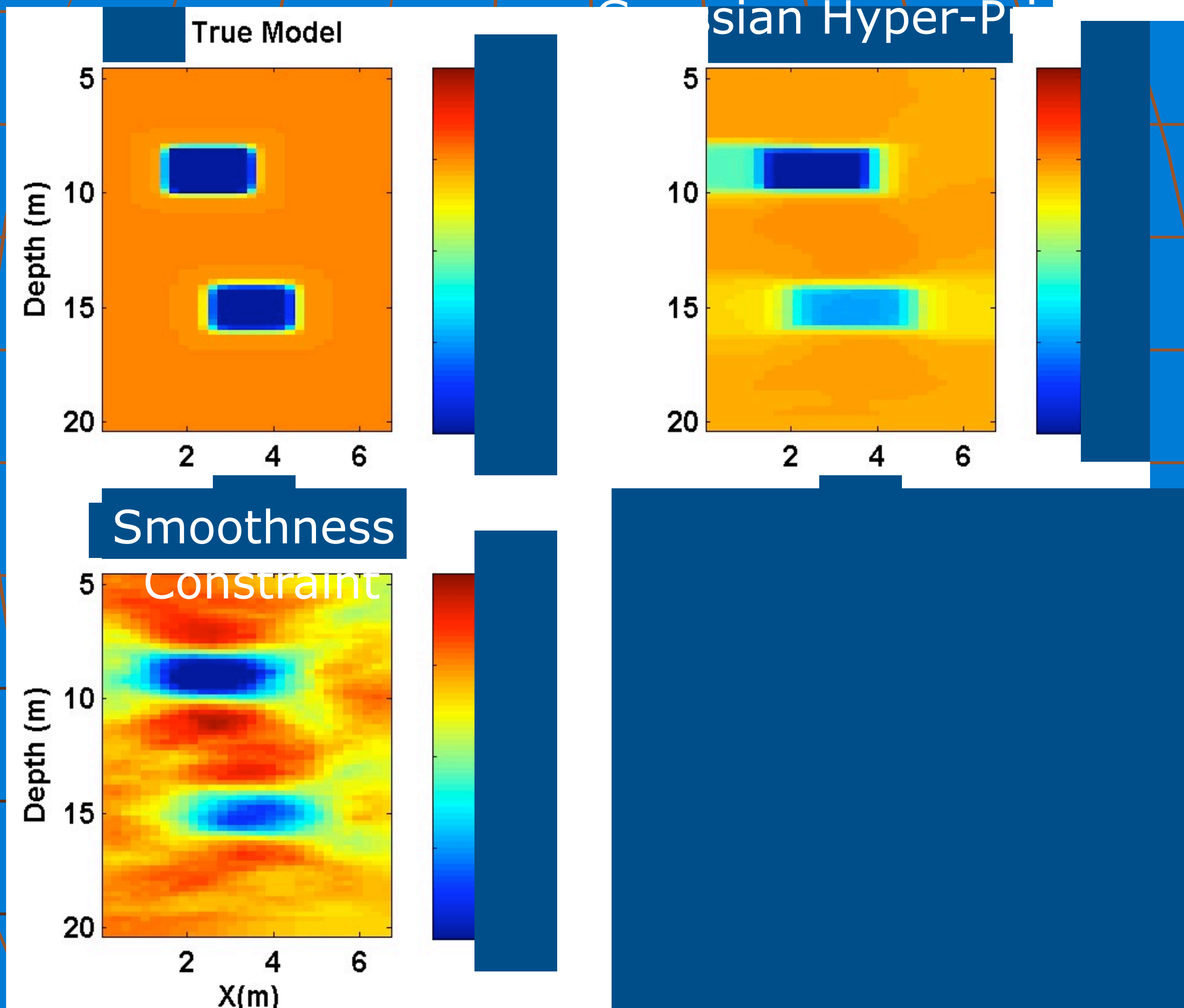
Evolution of Prior Distribution Parameters θ



Effect of Hyper-Parameter γ (Very Important)



2D Near-Surface Tomography Example



Comments

- Low-frequency prior + blockyness constraints → Deterministic inversion
- High frequencies are still missing

Deterministic Inversion

1. Deterministic seismic inversion is limited to the estimation of band limited (since seismic is band limited) reflectivity series which corresponds to blocky average impedance profile.

2. In deterministic inversion the estimation is trade-off between resolution and accuracy.

3. The missing low frequencies contain the critical information concerning the absolute values of impedance.

Stochastic Inversion

1. Stochastic seismic inversion is based on generating multiple equi-probable realizations of the model parameters dictated by the available log data and comparing the results with the observed data using forward modeling.

2. Stochastic impedance volume derives its areal resolution from the seismic data and vertical resolution from the log data used in inversion process.

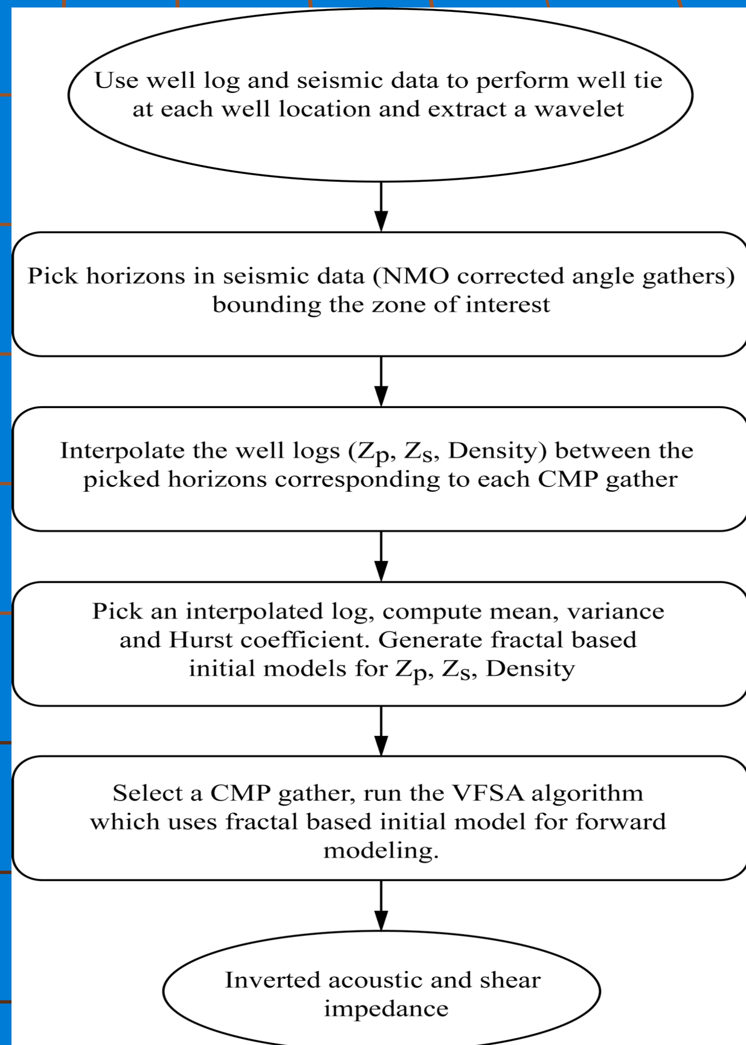
Need of geo-statistical inversion

To address the smoothing problem in deterministic inversion, we need to introduce an additional variation to our estimates which corrects the CDF.

Since geo-statistical simulation is not unique, there are many possible solutions which satisfy the data. Each possible solution is referred to as realization.

Francis, A., First Break, 2006

Our approach



Srivastava and Sen
2009 a, b

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Background of our strategy

- It is observed from the analysis of several horizontal and vertical well logs that porosity distributions follow fractional Gaussian noise (fGn) characteristic (Hewett,1986,SPE; Hardy,1992,SPE).
- A time/space series is said to follow fGn characteristic if its statistical measures exhibit following behavior:

- Spectral density of fGn follows power law with a scaling exponent (alpha):

  $S(\omega) = \lambda|\omega|^{\alpha}$

- Variogram follows power law in terms of intermittency coefficient or Hurst coefficient (H) as:

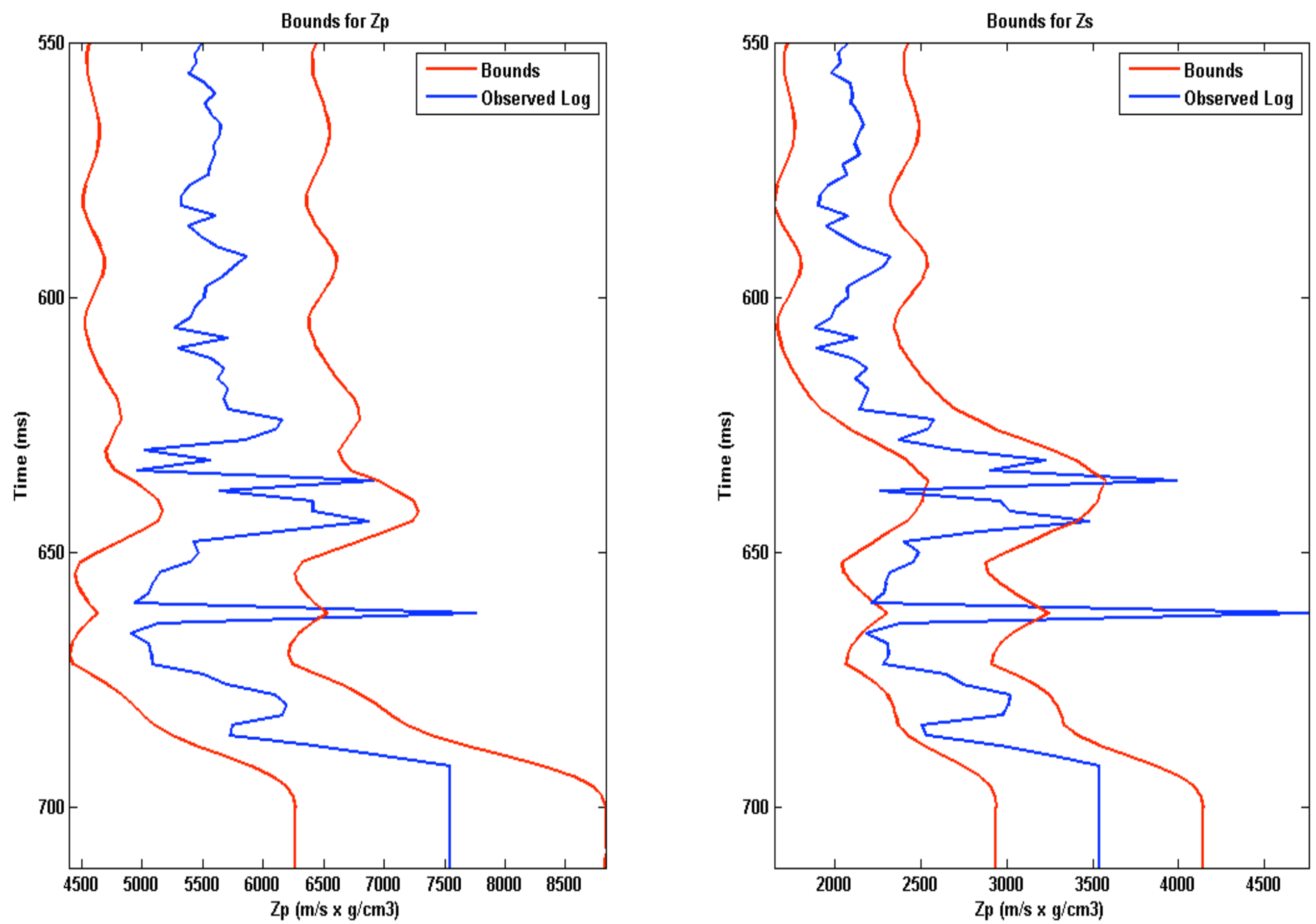
  $\gamma\ (\tau) = a - b\ |\ \tau\ |^{2H-2}$

- Co-variance also follows power law with H

  $Cov(\tau) = \sigma^2/2\ [\ |\tau+1|^{2H} - 2|\tau|^{2H} + |\ \tau-1|^{2H}]$

Pre-Stack Inversion

Zp & Zs bounds



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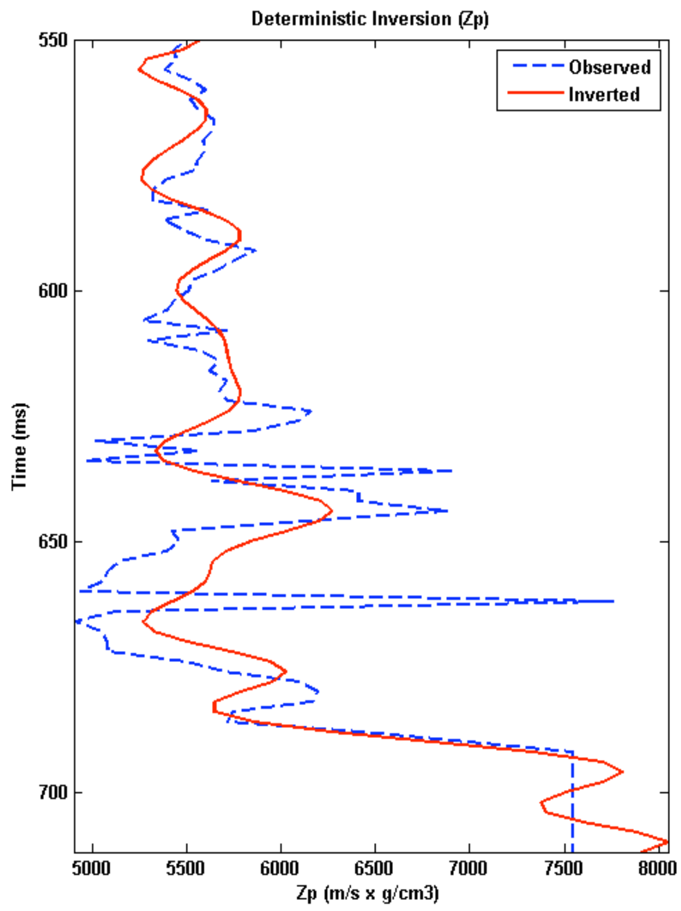
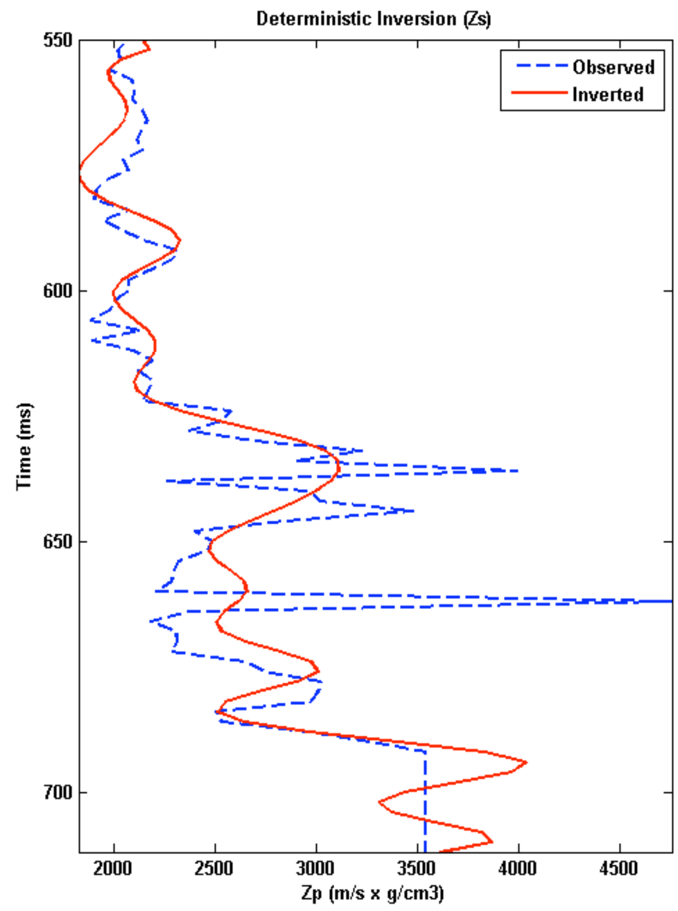
Deterministic Zp & Zs inversion at well location





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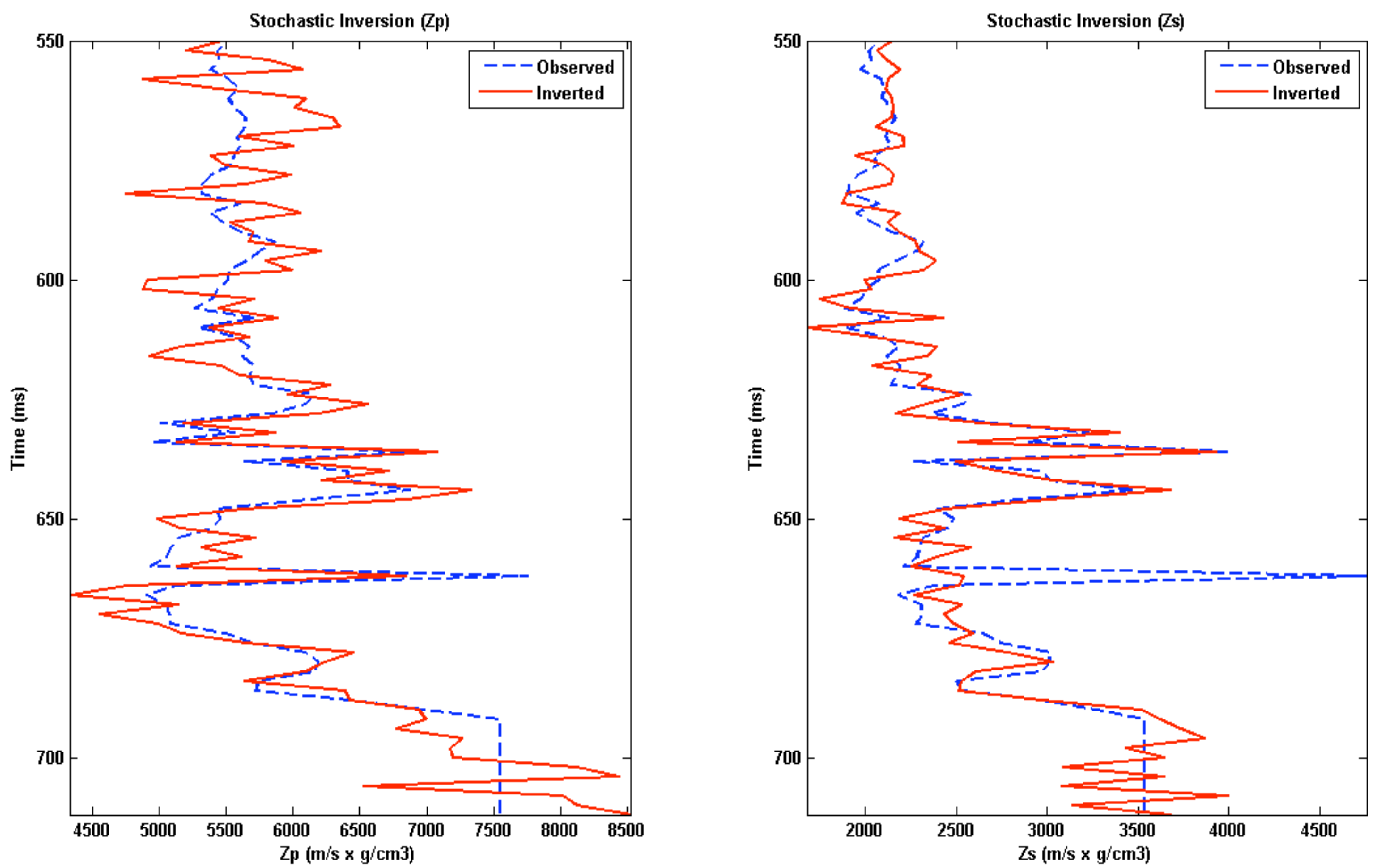
Stochastic Zp & Zs inversion at well location



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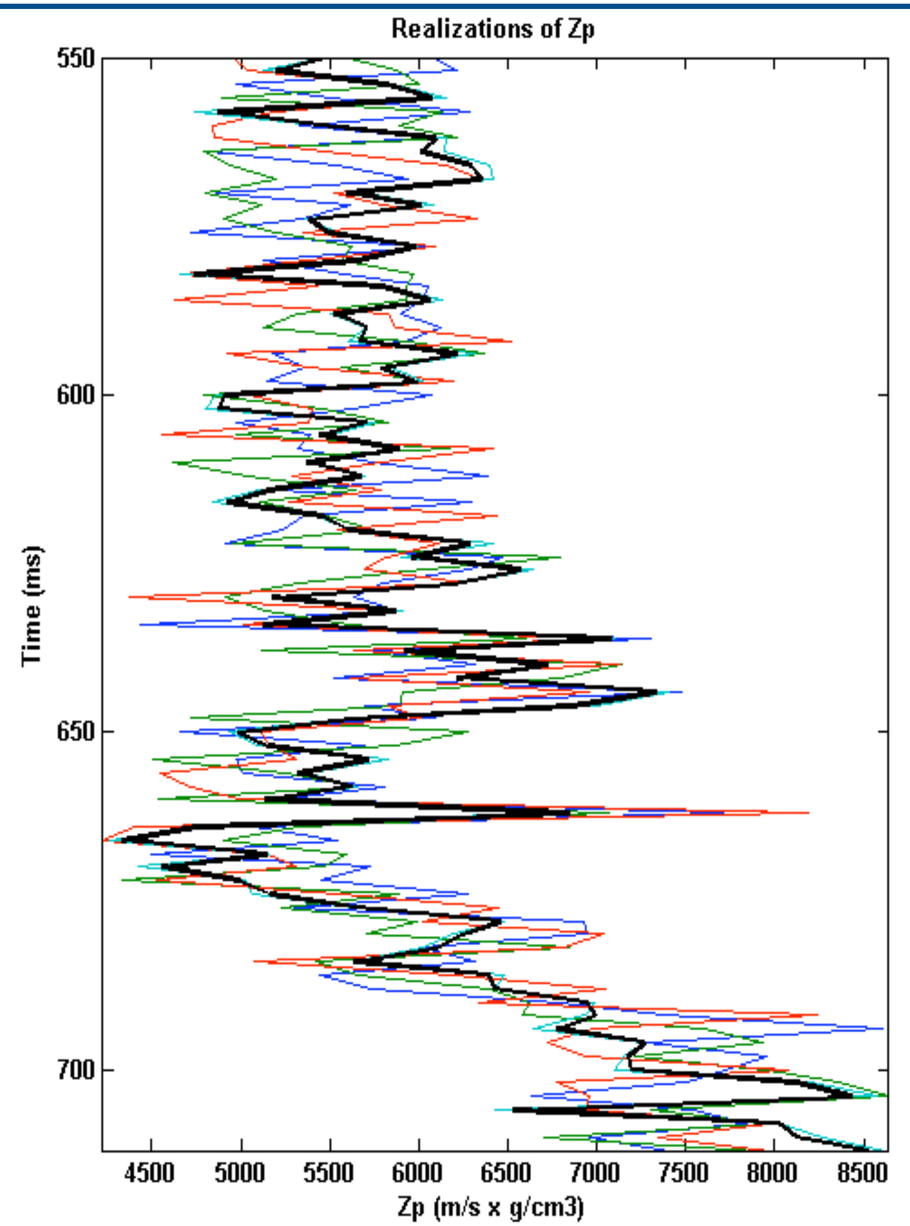
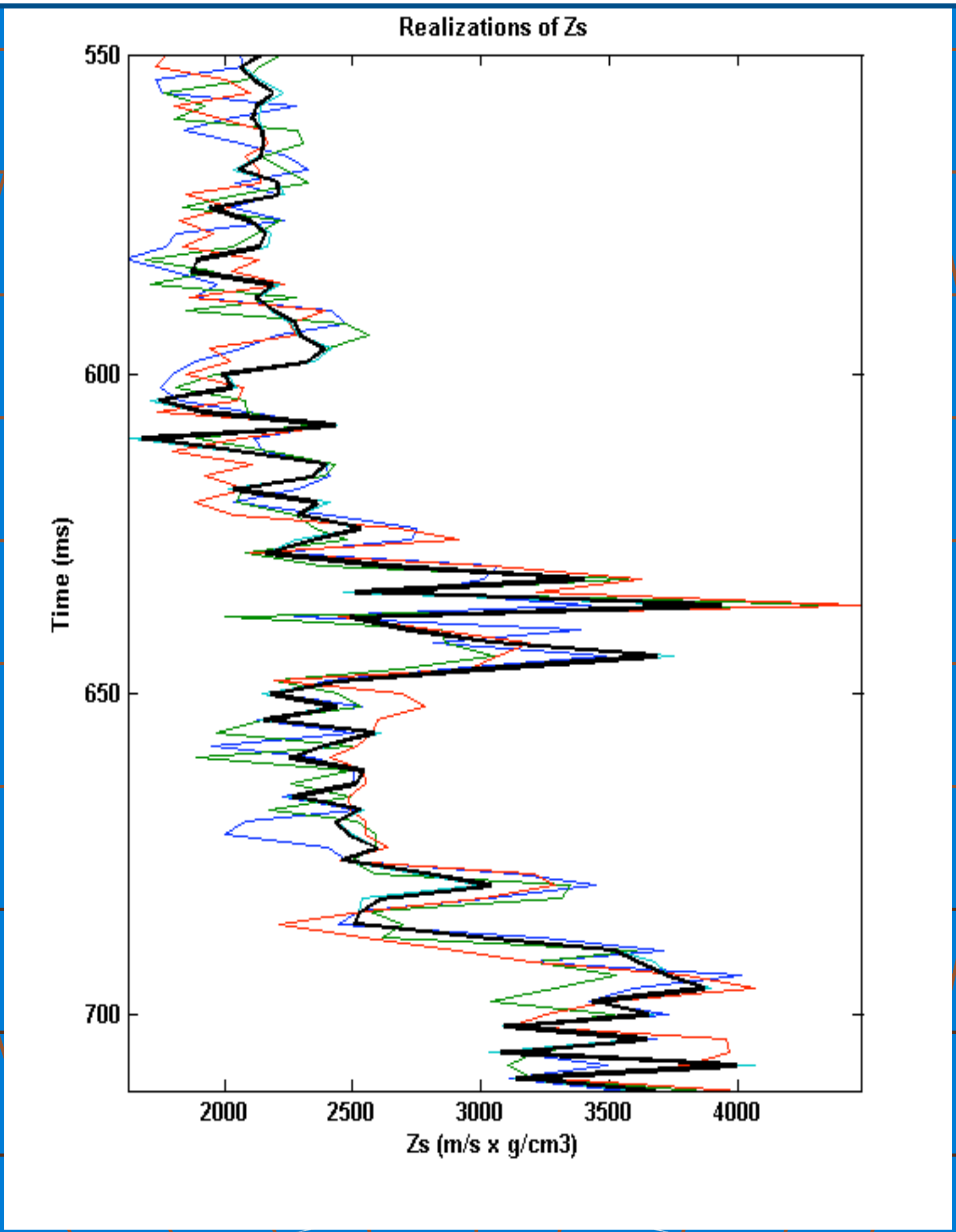
25 realizations of Zp (one trace is shown)

Black line shows mean of realizations

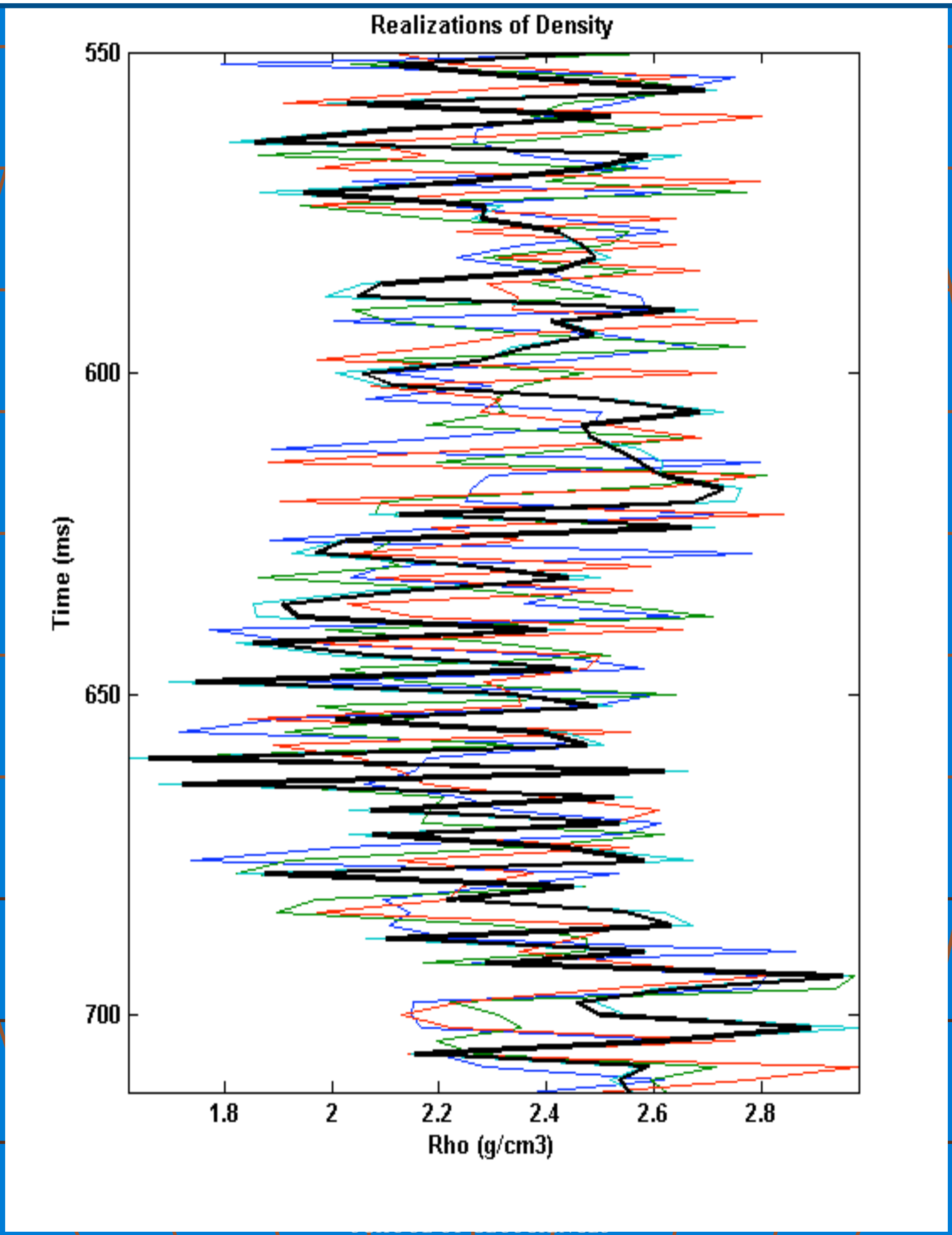


25 realizations of Zs (one trace is shown)



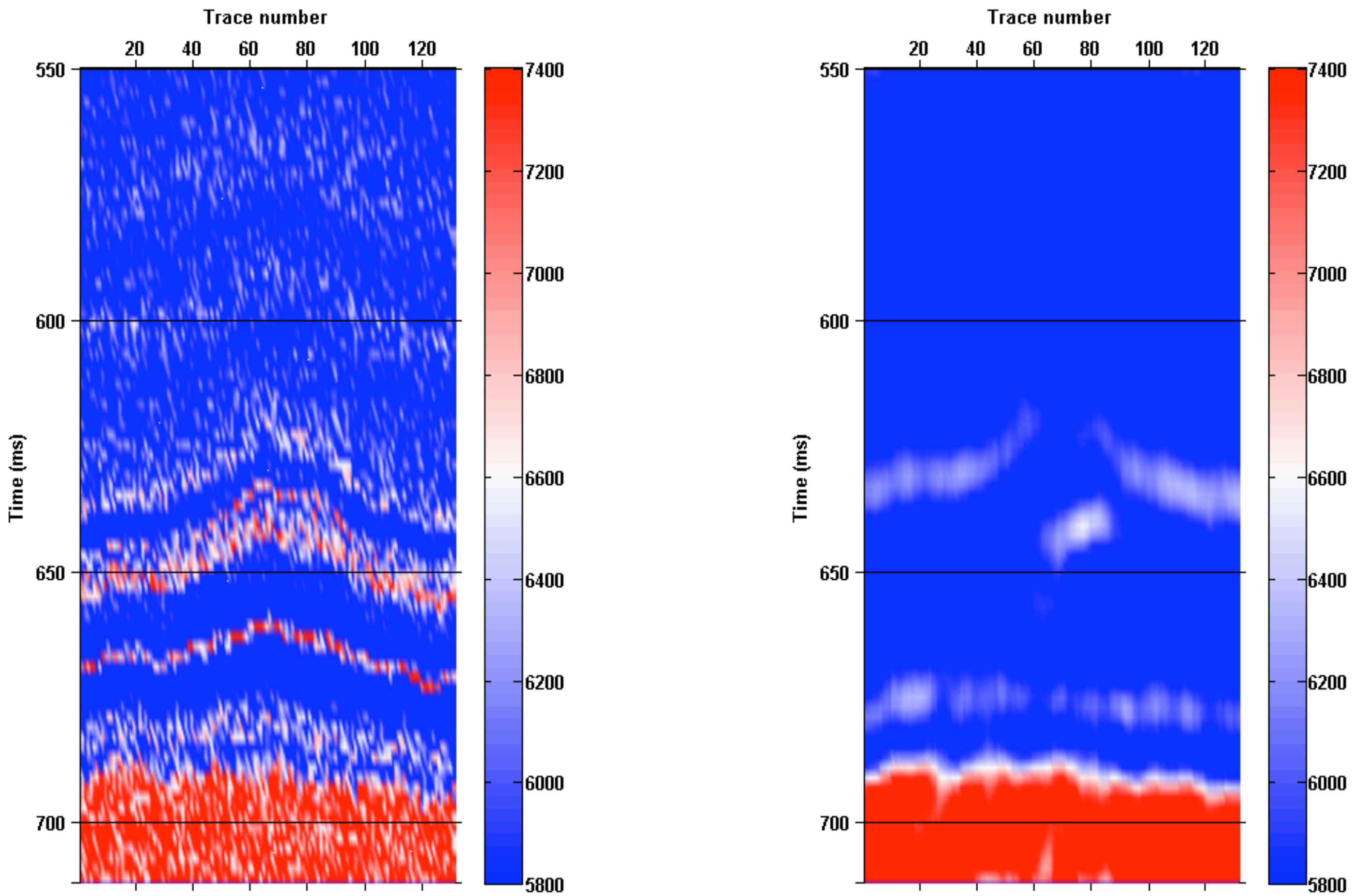
Black line shows mean of realizations

25 realizations of density (one trace is shown)



Black line shows mean of realizations

Pre –stack stochastic and deterministic results for **Zp** in same scale for a line



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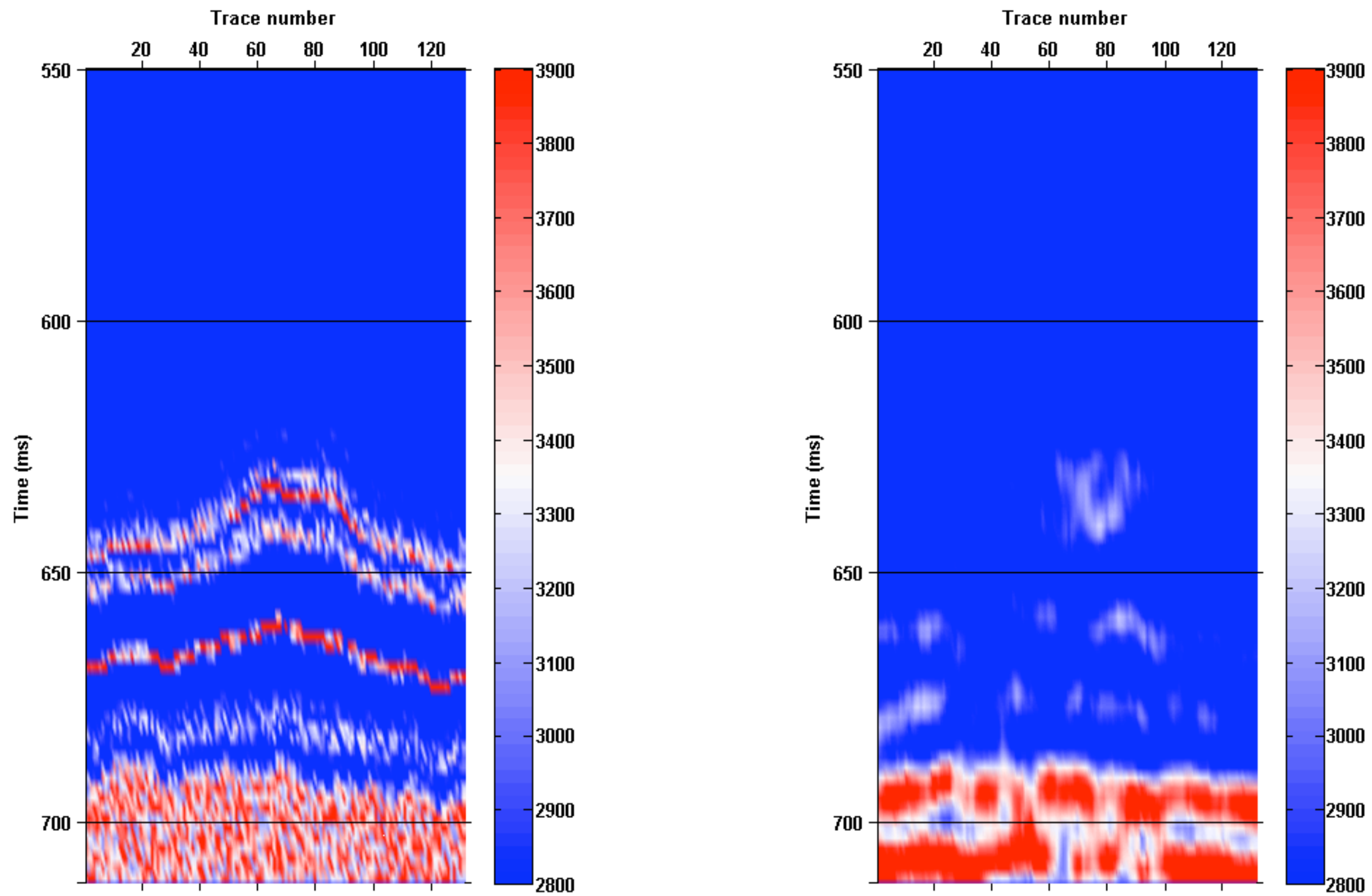
Pre –stack stochastic and deterministic results for **Zs** in same scale for a line



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Summary of stochastic inversion

1. Fractal based prior gives good starting solution and its efficient to generate such prior.
2. This method provides the realistic frequency band in prior model close to those available in the log data.
3. Results in high resolution estimates of the model parameters.
4. Noisy characteristic in estimates could be result of 1D modelling which can be circumvented using 2D initial model based on fractals.

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Challenges

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- Beyond 1D – null space is not well understood
- Low frequency problem – what if there are no well logs?

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An example of 2D inversion

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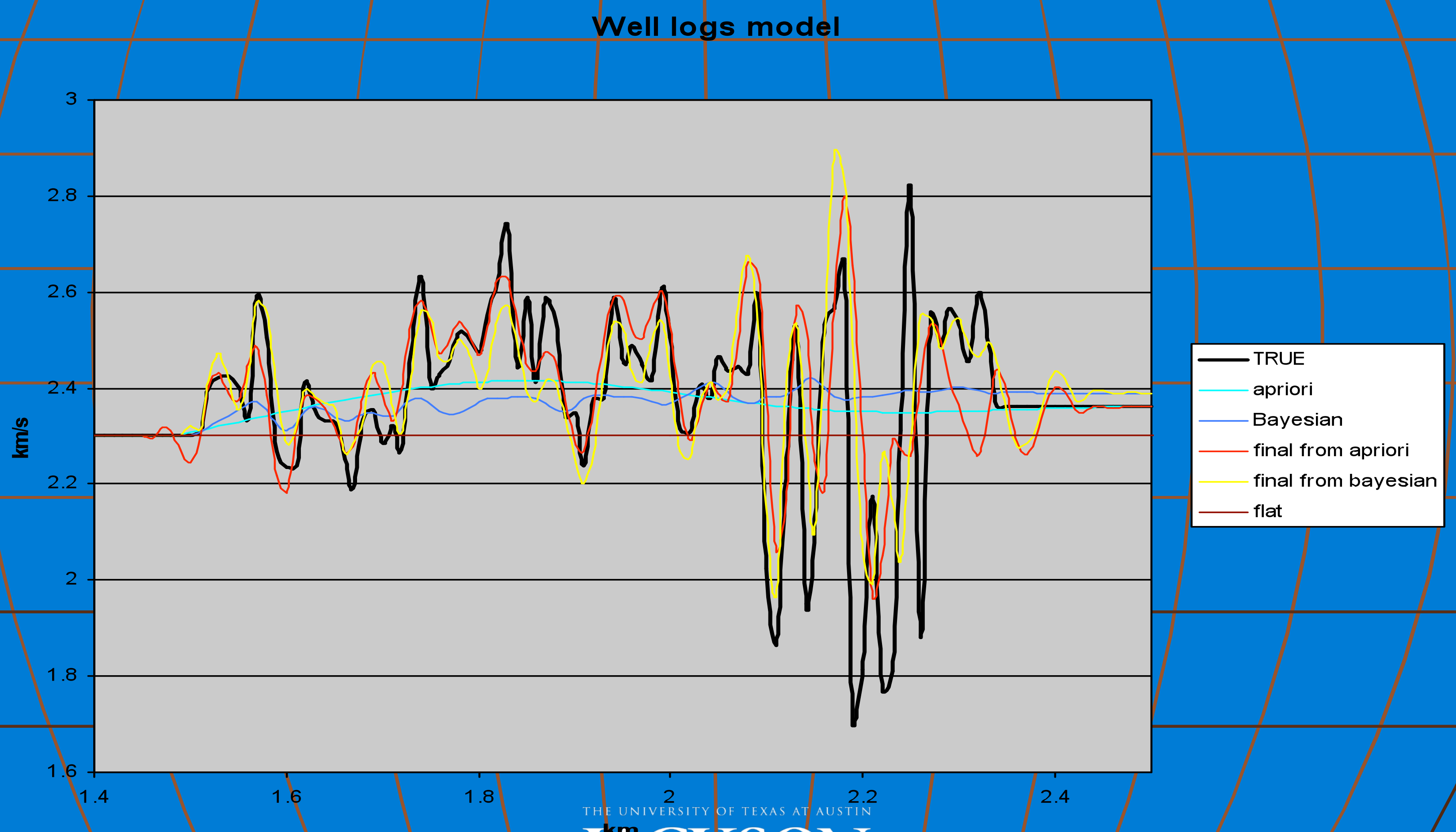

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Thank You

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