



Free-surface Multiple Attenuation Using Inverse Data Processing in the Plane Wave Domain

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JACKSON

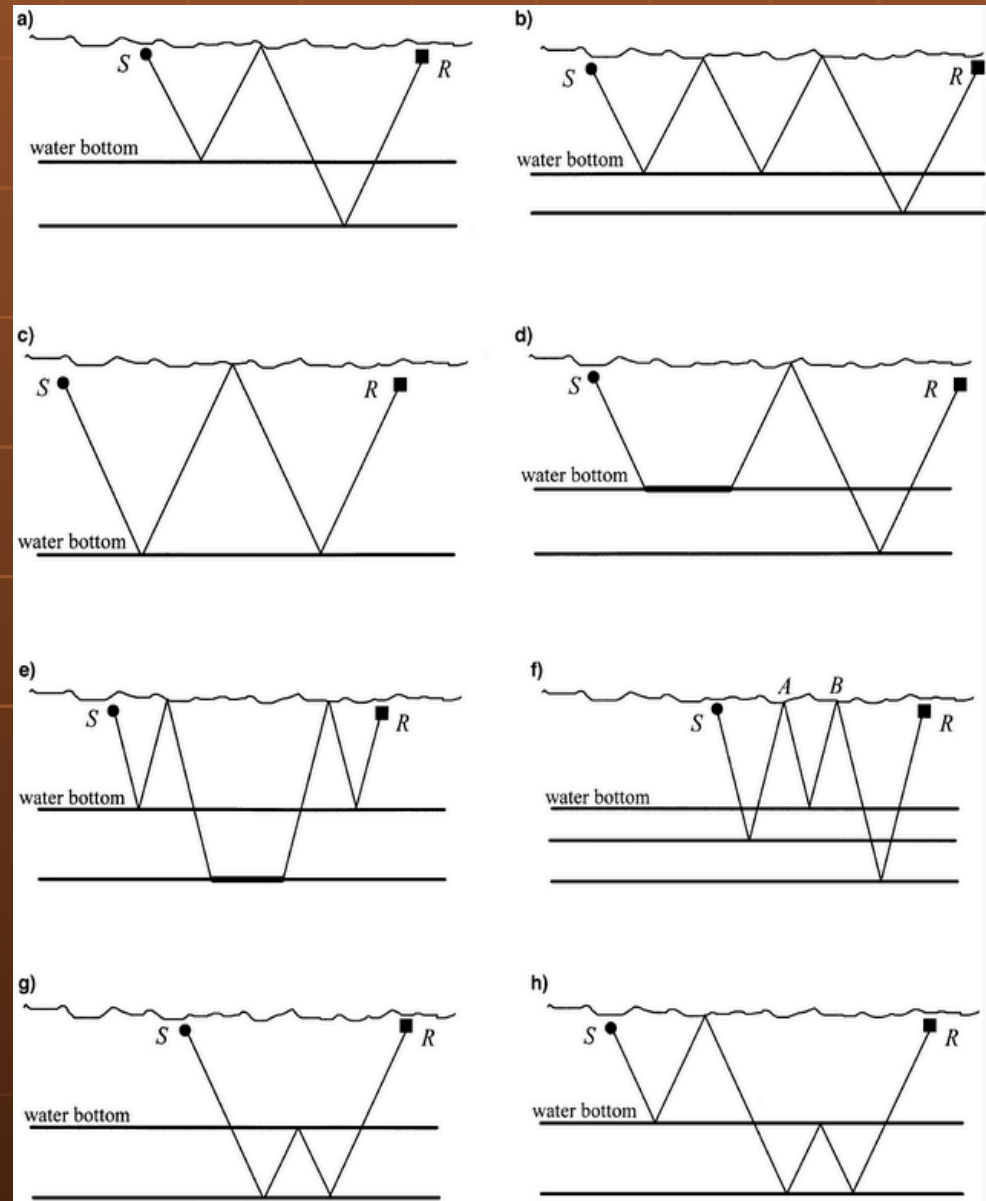
SCHOOL OF GEOSCIENCES

Outline

- Introduction to Multiple Attenuation
- Inverse data processing theory
- Inverse data processing in the coupled plane wave domain
- A field data example
- Discussion and conclusions

Introductio

Typical Multiples



Reference:

Dragoet, W.H. and Jericevic, Z., 1998, Some remarks on surface multiple attenuation:

Geophysics, 63, 772-789.

Introduction



Multiple
Attenuation
methods

n

Filtering method (**Assumptions**)

periodicity, separability

Predict deconvolution, stack, F-K
filtering, Radon transform

Prediction-subtraction method (**No
assumptions, acquisition information required**)

Wavefield extrapolation -- Wiggins, 1988

Free surface multiple attenuation:

Feed back method: Verschur, 1992

Inverse-scattering method: Weglein, 1997.

Invariant embedding technique: Sen, 1998; Faqi Liu, 2000

Introduction



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Prediction subtraction method

Predict multiples well, subtraction may damage primary energy.

Introduce adjacent traces to constrain the subtraction:

Monk, D. J., Constrained cross-equalization, 1993

Spitz, S, Pattern recognition, 1999

Wang Y. Expanded multichannel matching, 2003

Lu, W, Independent component analysis, 2006

Li Peng, pseudomultichannel matching, 2007

Fomel Sergey, regularized nonstationary regression, 2008

.....

Introduction



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Inverse data processing

- A development of **prediction-subtraction** theory (Mainly SRME).
- Can **separate** multiples and primaries in a very natural way.
- In inverse data space (**IDS** v.s. Forward data space **FDS**), **multiples** will be focused **around zero time and offset**, while **primaries** are at **negative time**.

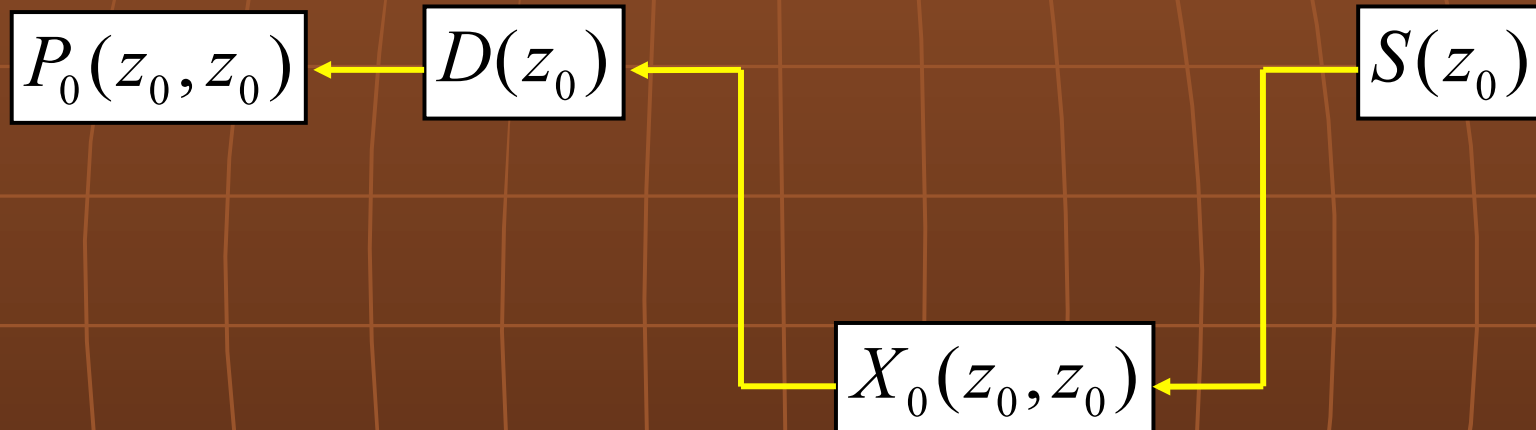
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IDP in x-t domain

SRME theory: Feedback model without the free surface



$$P_0(z_0, z_0) = D(z_0)X_0(z_0, z_0)S(z_0)$$

$S(z_0)$ source function

$X_0(z_0, z_0)$ primary

reflectivity matrix

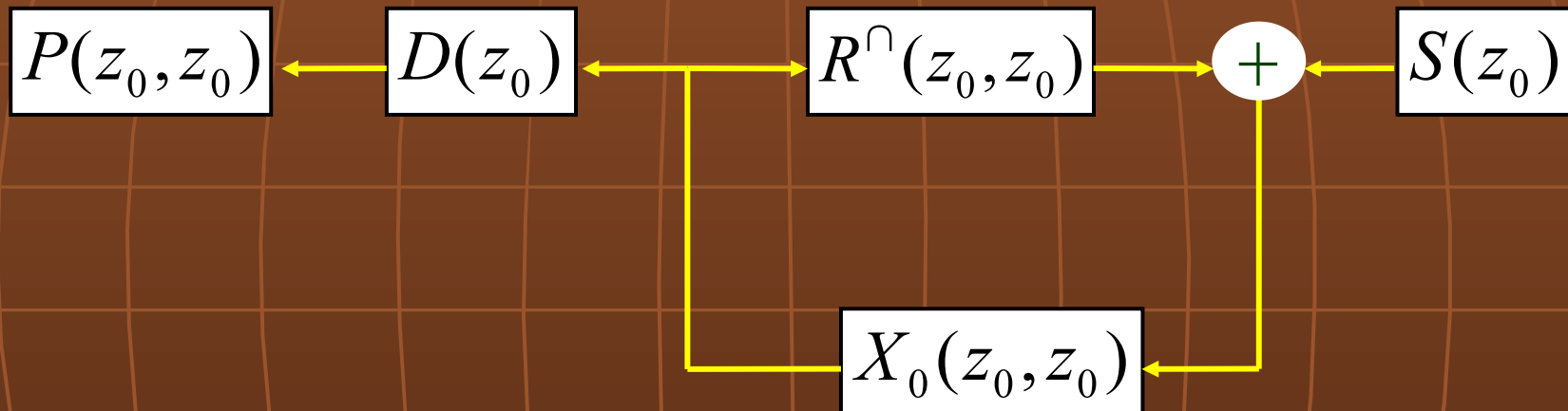
$D(z_0)$ detector property matrix

$P_0(z_0, z_0)$ wavefield without

IDP in x-t

domain

SRME Theory: Feedback model with the free surface



$$P = P_0 + (P_0 A)P_0 + (P_0 A)^2 P_0 + \dots$$

$$P_0 = DX_0 S$$

$$A = S^{-1} R^\wedge D^{-1}$$

R^\wedge free surface
reflectivity

P wavefield with free
surface effect ¹⁰

A surface operator

IDP in x-t domain

Inverse data processing in the x-t domain (Berkhout, 2006)

$$P = P_0 + (P_0 A)P_0 + (P_0 A)^2 P_0 + \dots$$

$$P = P_0 + (P_0 A)P$$

$$P_0 = (I - P_0 A)P$$

$$P = [I - P_0 A]^{-1} P_0$$

$$P^{-1} = P_0^{-1} [I - P_0 A]$$

$$P^{-1} = P_0^{-1} - A$$

P_0^{-1} primaries with time information

located at negative times

A surface operator without time information

located around zero time

Requirement for the data (Same as SRME)

1. Full wavefield data upto zero offset - extrapolation
2. Regular geometry, equal distance between shot and receiver points -

Reference :

Berkhout, A. J., 2006, Seismic processing in the inverse data space: *Geophysics*, 71, no. 4, A29–A33.

IDP in x-t domain

Advantages

1. Very simple compared with multiple elimination in the forward space;
2. Does not harm any primary energy;

Key point: Matrix Inversion Least squares inversion
and SVD inversion

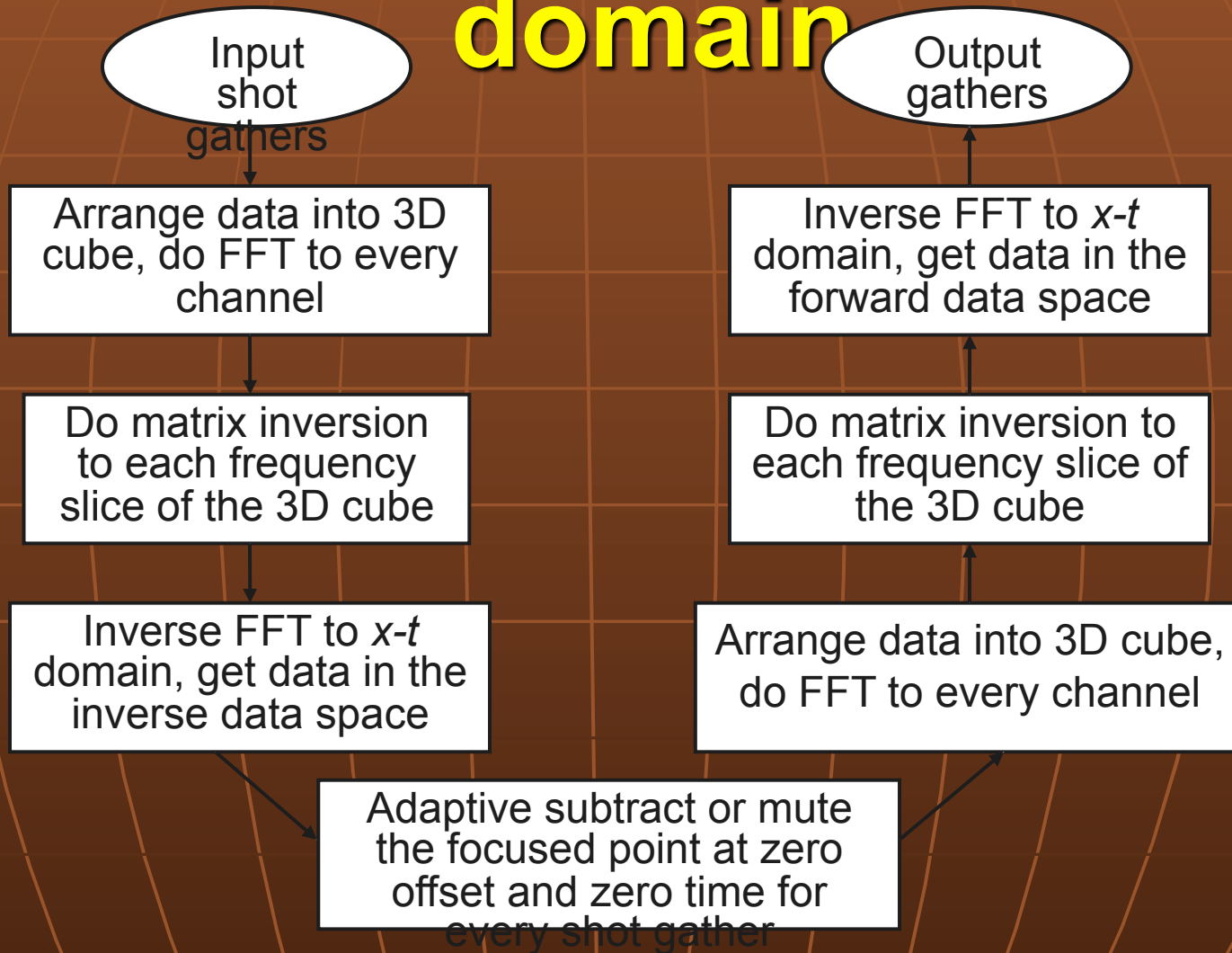
$$\begin{cases} P^{-1} = BP^H \\ B = (P^H P + \varepsilon^2 I)^{-1} \end{cases}$$

$$\begin{cases} P = U[\text{diag}(\omega_j)]V^T \\ P^{-1} = V[\text{diag}(1/\omega_j)]U^T, j < N \end{cases}$$

First two eqations from:

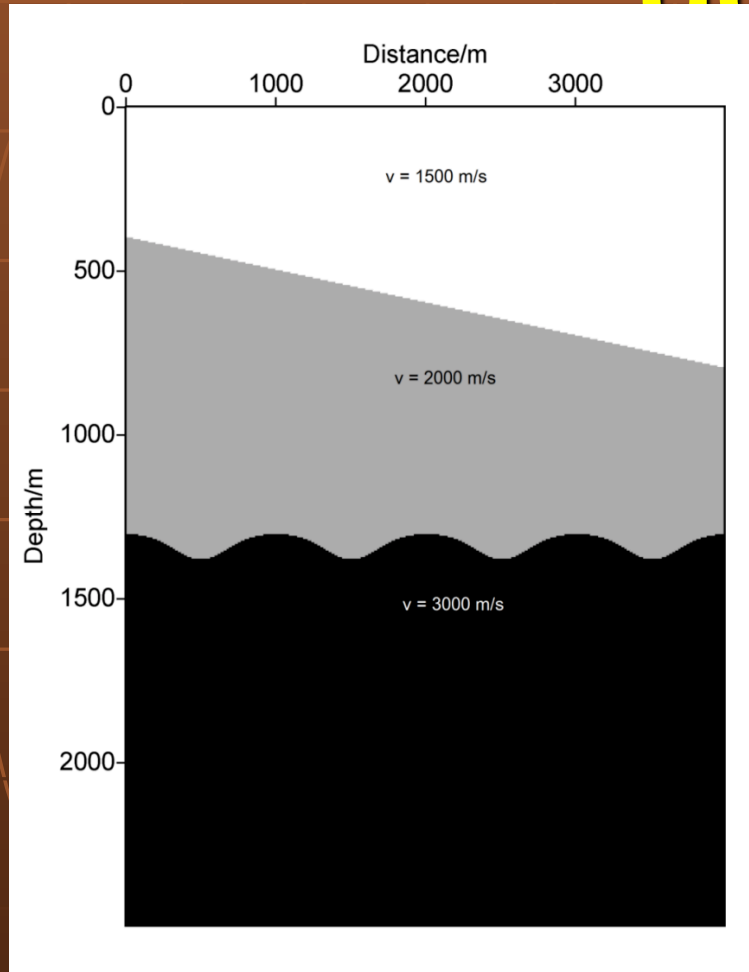
Berkhout, A. J., and D. J. Verschuur, 2006, Focal Transform, an imaging concept for signal restoration and noise removal: *Geophysics*, 71, no. 6, A55–A59.

IDP in $x-t$ domain

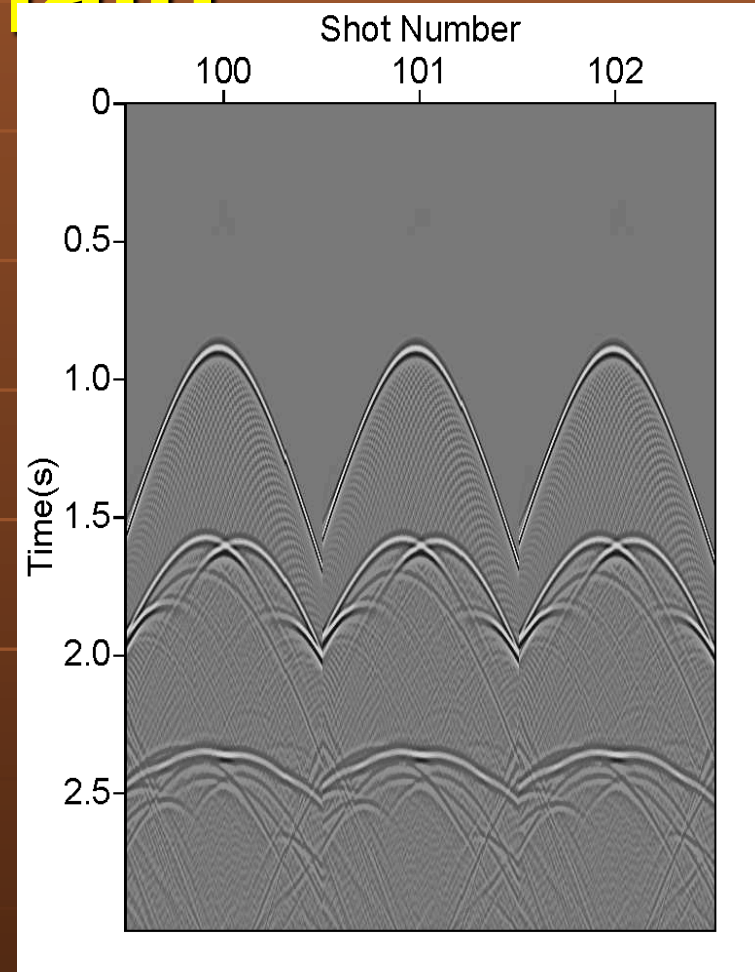


Workflow of $x-t$ domain inverse data processing

IDP in x-t domain

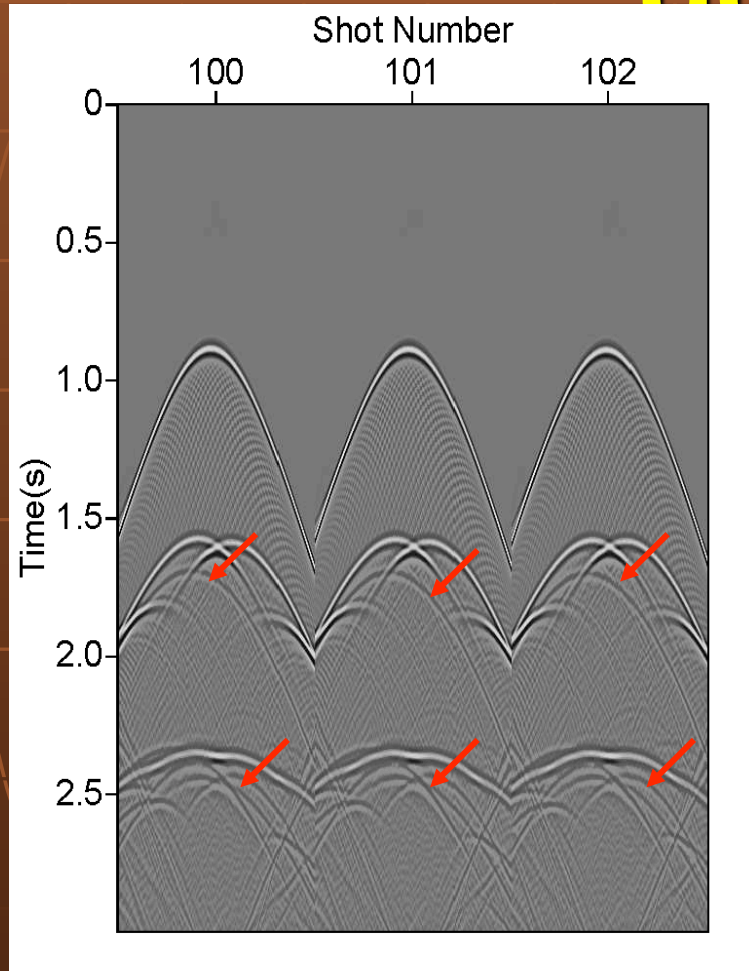


Velocity model

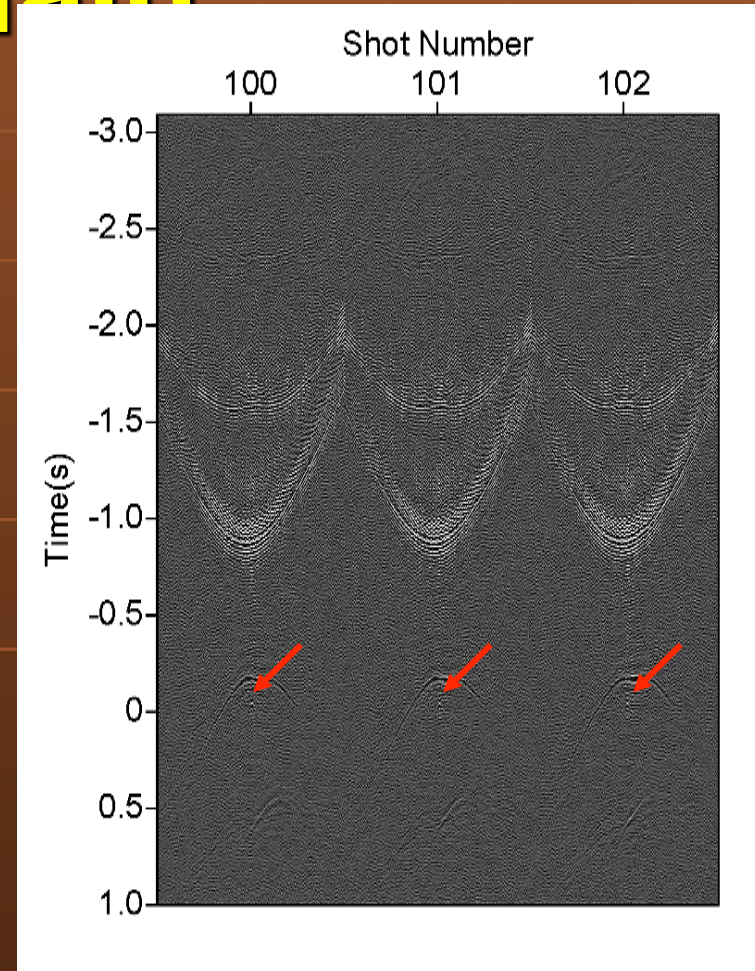


Three typical shot gathers ¹⁴

IDP in x-t domain

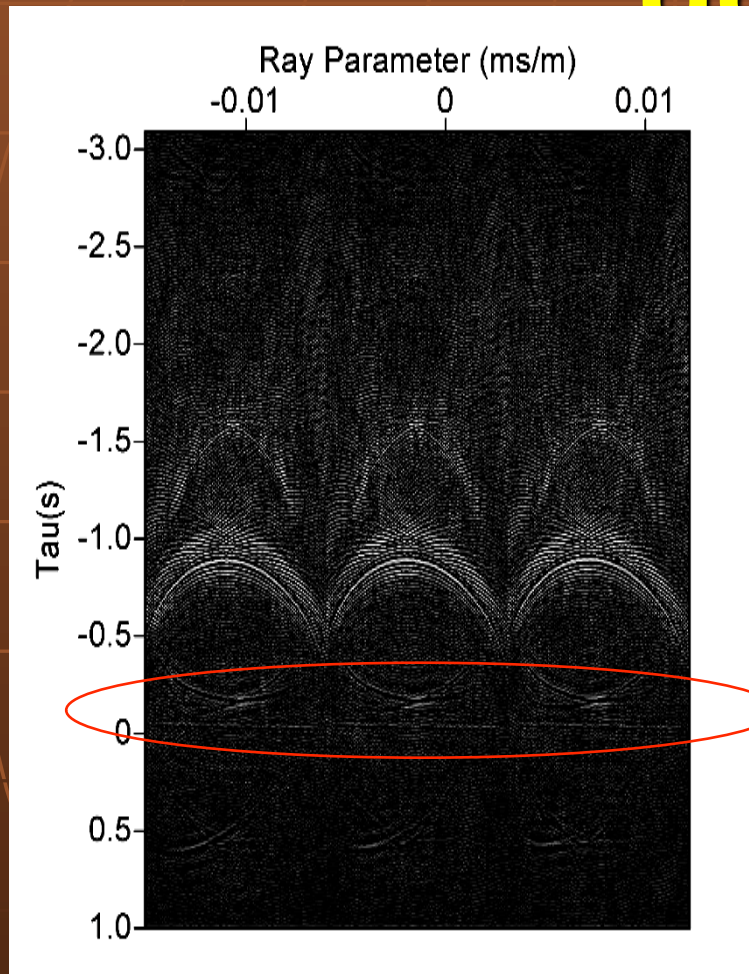


Three typical shot gathers

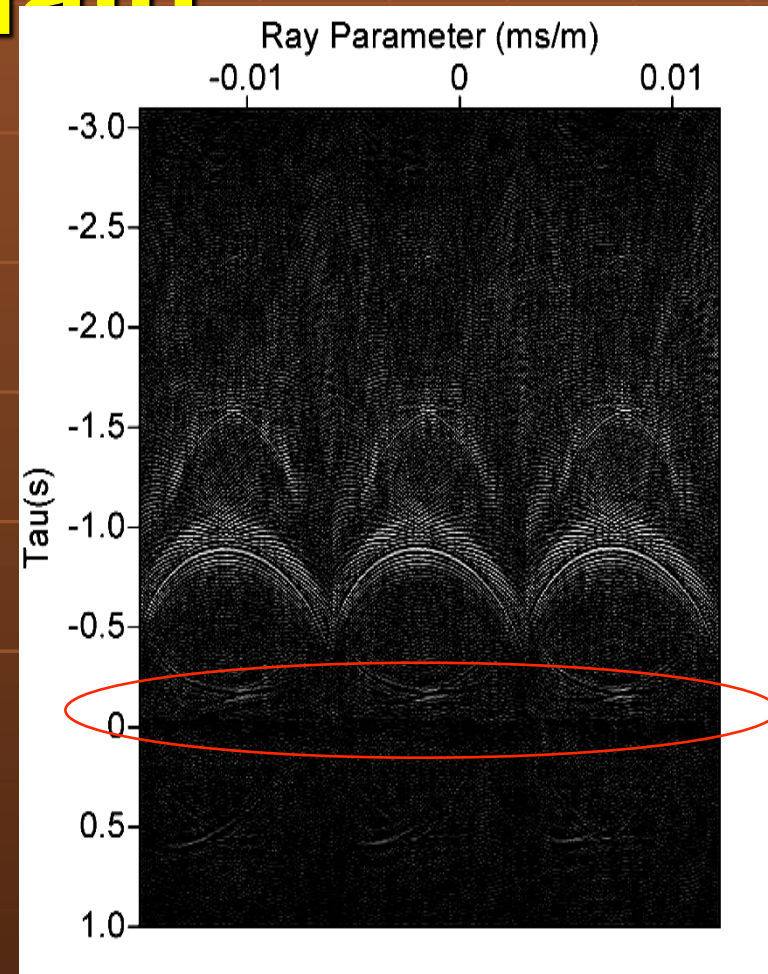


Data in the inverse data space¹⁵

IDP in x-t domain

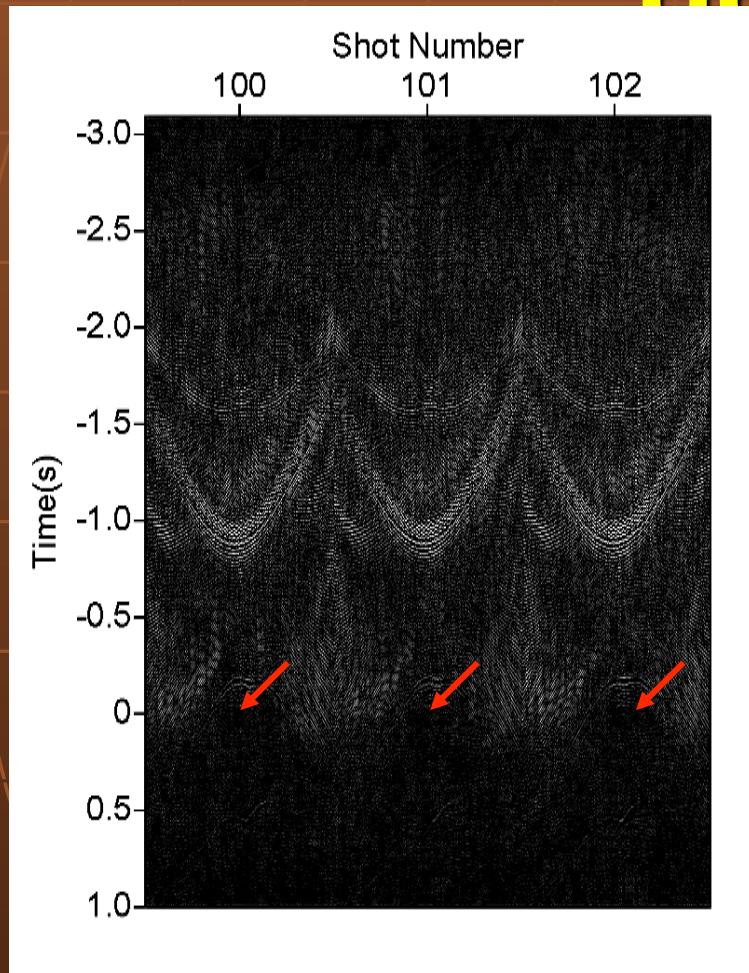


Tau-p transform of inverse data

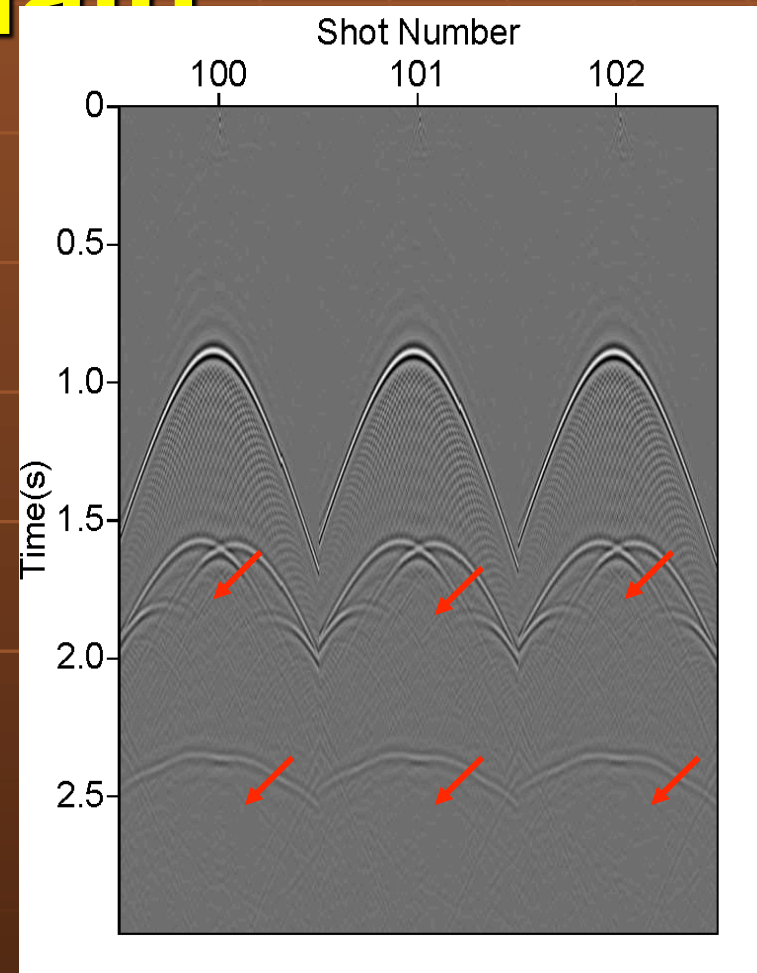


Adaptive subtraction of the line

IDP in x-t domain

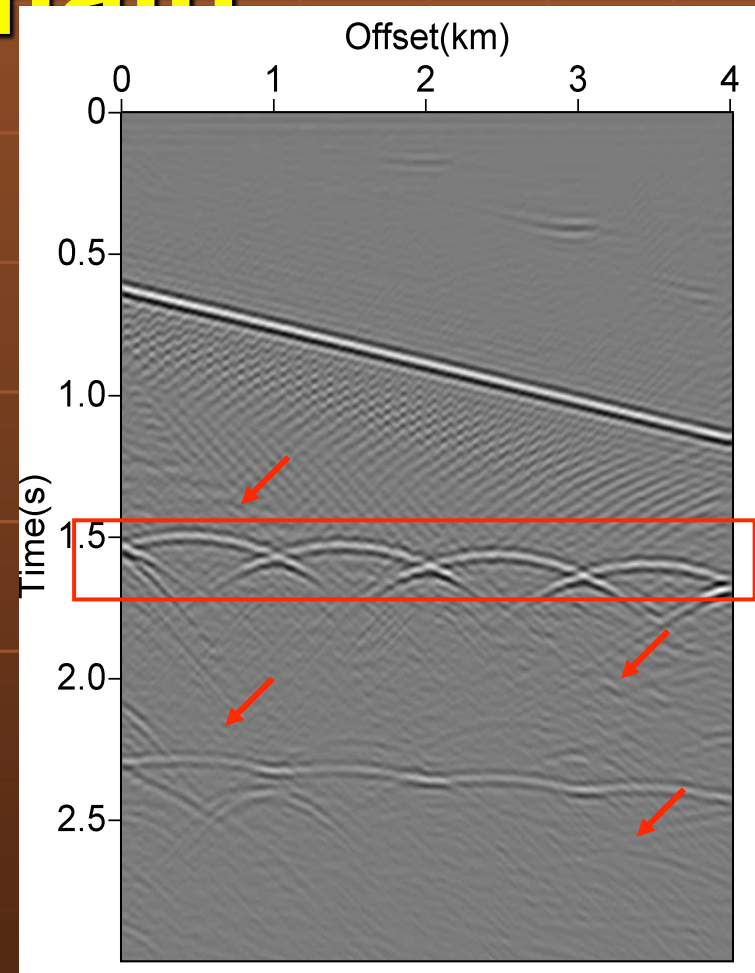
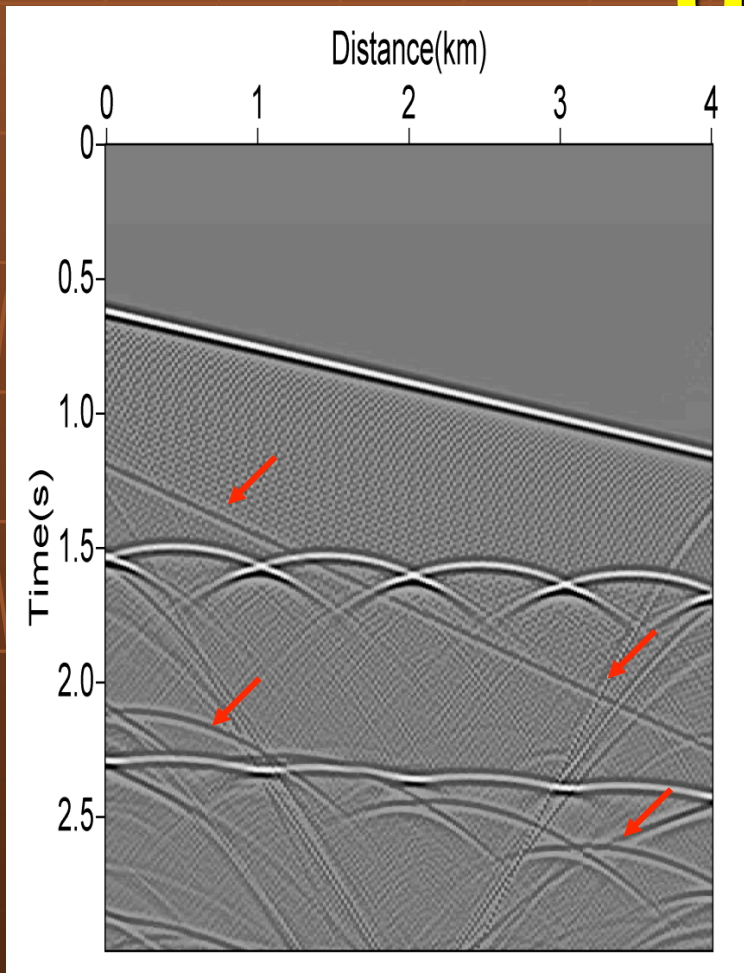


Data in the IDS after point muting



Data transformed back ¹⁷

IDP in x-t domain



Zero offset comparion before and after inverse data processing

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IDP in plane wave domain

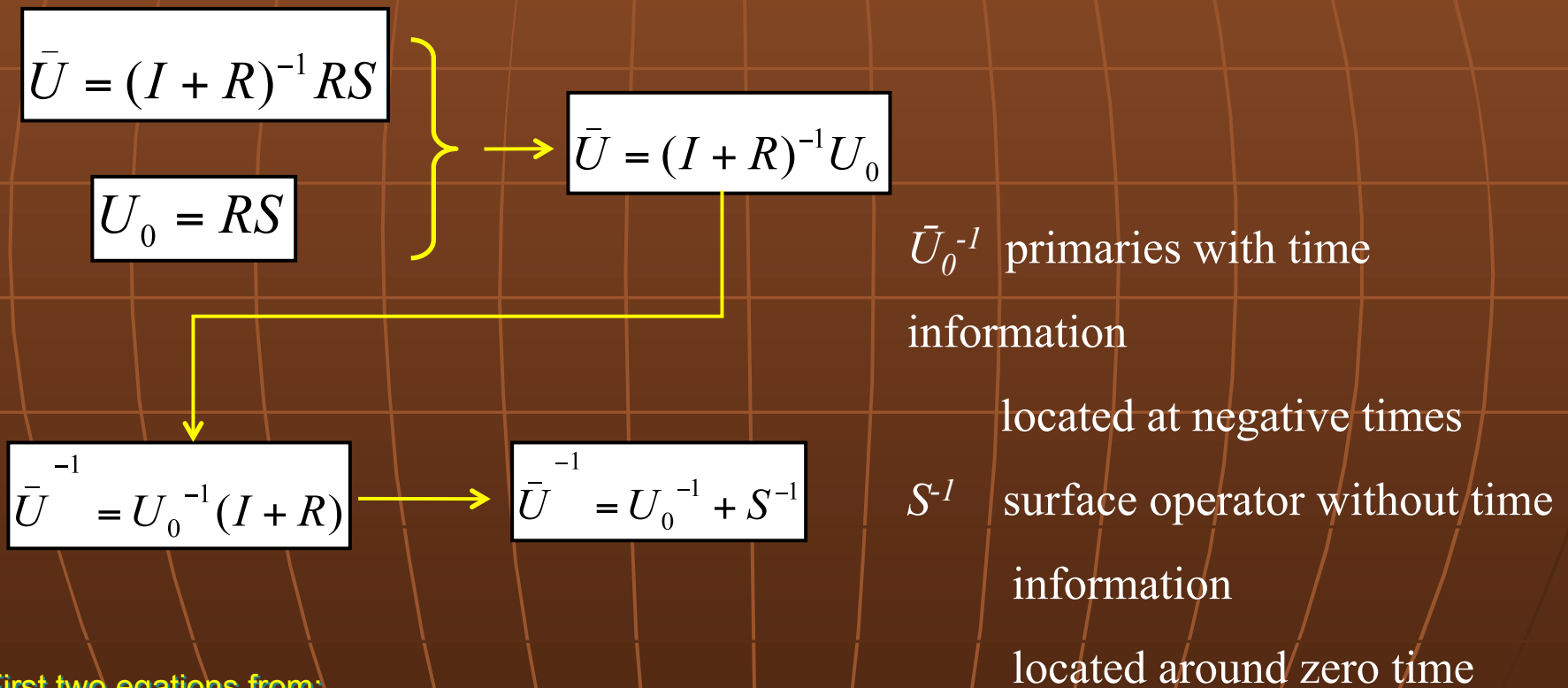


Motivation:

- Infinite relationship between primaries and multiples can not be satisfied;
- Matrix inversion introduce noise and artifacts;
- Tau-p transformation can compress seismic data, while focus seismic energy around the main diagonal, which will stabilize the inversion.

IDP in plane wave domain

Inverse data processing in the plane wave domain (Ma et al, 2009)



First two equations from:

Faqi Liu, Mrinal K. Sen and Paul L. Stoffa, 2000, Dip selective 2D multiple attenuation in the plane wave domain, *Geophysics*, 65(1), 264~274

IDP in plane wave domain



Advantages compared with IDP in $x-t$ domain

1. Compress seismic data using Tau-p transform, reduce computation cost;
2. Focus energy around the diagonal, stabilize the inversion.

Key point: Matrix Inversion

and SVD

$$\begin{cases} \bar{U} = B\bar{U} \\ B = (\bar{U}^H \bar{U} + \varepsilon^2 I)^{-1} \end{cases}$$

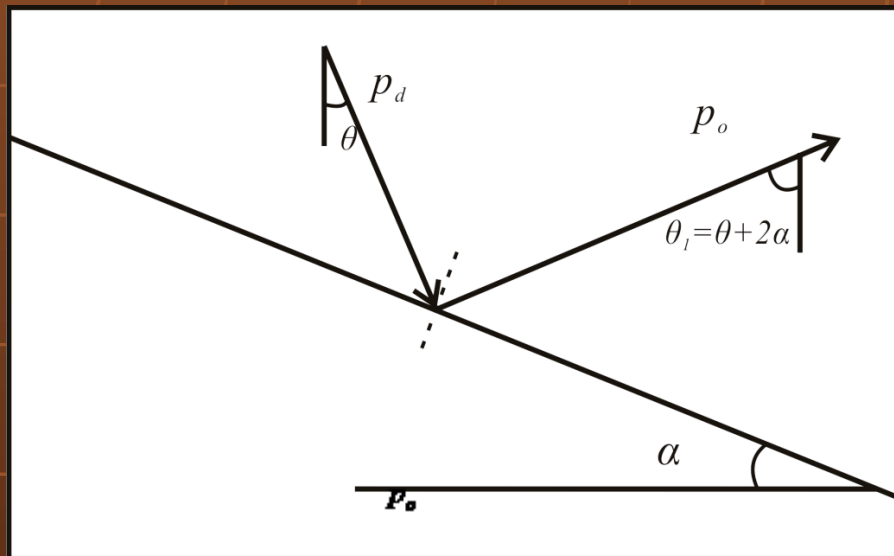
$$\begin{cases} \bar{U} = U[\text{diag}(\delta_j)]V^T \\ \bar{U}^{-1} = V[\text{diag}(1/\delta_j)]U^T, j < N \end{cases}$$

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Berkhout, A. J., and D. J. Verschuur, 2006, Focal Transform, an imaging concept for signal restoration and noise removal: *Geophysics*, 71, no. 6, A55–A59.

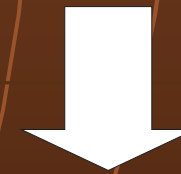
Plane-wave Domain Theory

Feature : Band-limited matrix (Liu et al, 2000)



Geometry of an incident and reflected plane waves at a dipping interface

$$p_o = \frac{\sin(\theta + 2\alpha)}{v} = p_d \cos 2\alpha + q_d \sin 2\alpha$$



$$|p_o - p_d| \leq \frac{2 \sin \alpha}{v}$$

$p_d = \sin \theta / v$
parameter

downgoing plane wave ray

$q_d = \sqrt{1/v^2 - p_d^2}$
parameter

upgoing plane wave ray
vertical slowness of incident ray

Plane-wave Domain Theory



Linear mapping relationship (Liu et al, 2000)

Data in the source-offset coordinates v.s. plane wave data

$$d(x_s, x, t) = \frac{\omega^2}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} D_{pl}(p_d, p_o, \omega) e^{i\omega p_o x} e^{i\omega(p_o - p_d)x_s} dp_o dp_d$$

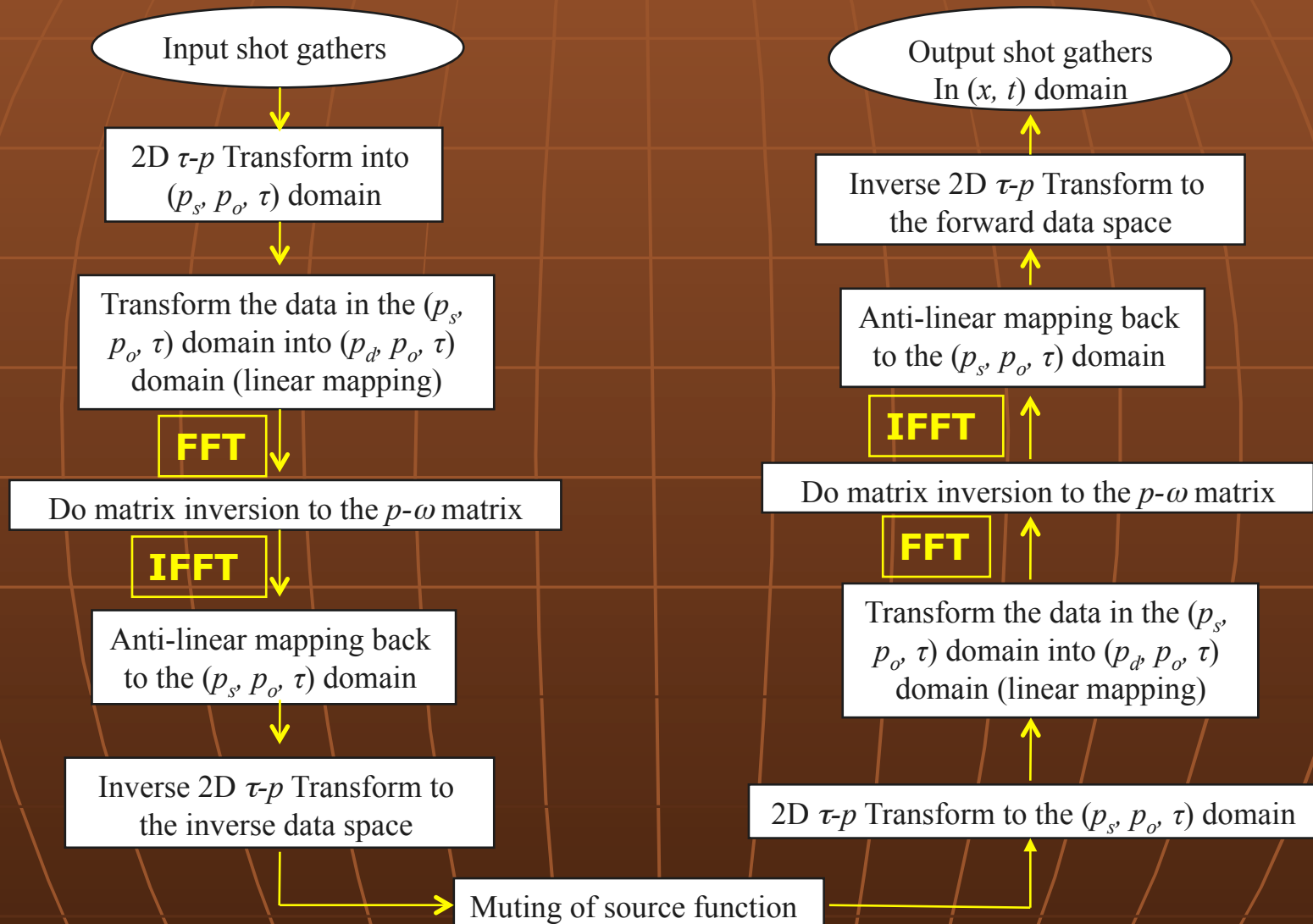
Inverse 2D t - p transform

$$d(x_s, x, t) = \frac{\omega^2}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} D_{tp}(p_s, p_o, \omega) e^{i\omega p_o x} e^{i\omega p_s x_s} dp_s dp_d$$

Linear mapping

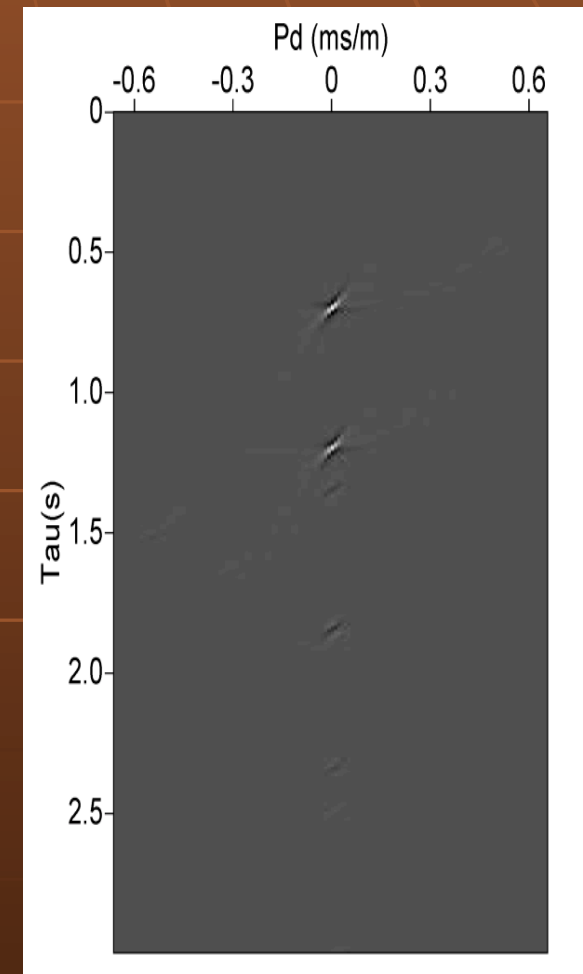
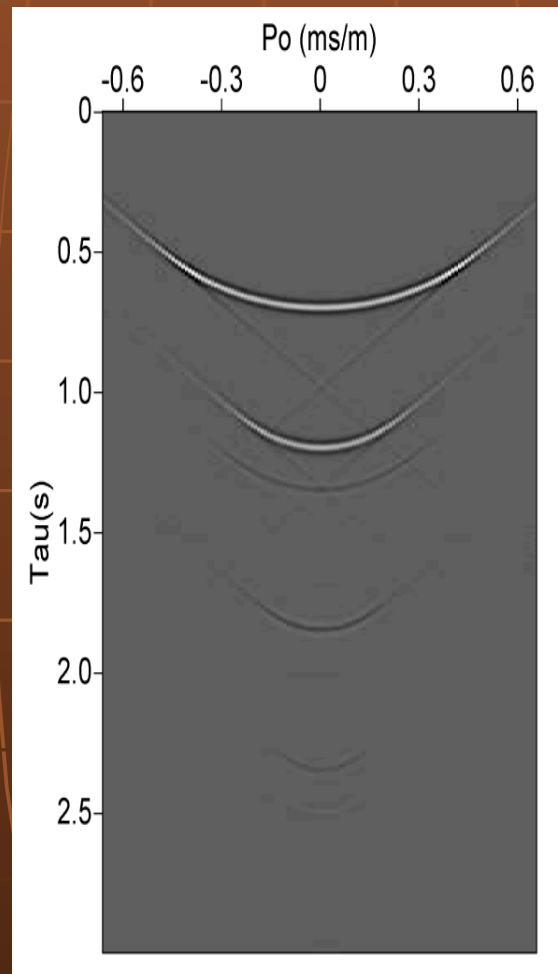
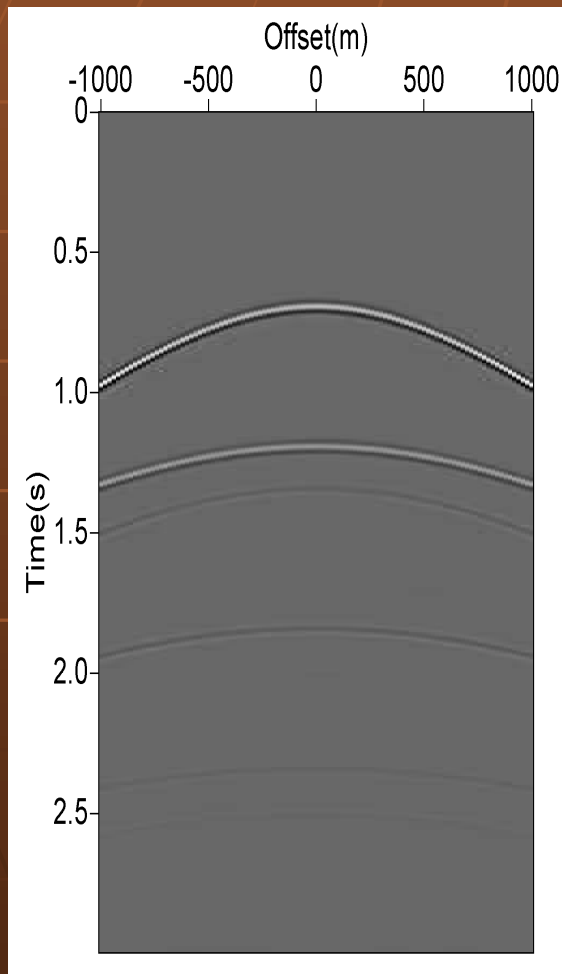
$$D_{pl}(p_d, p_o) = D_{tp}(p_o - p_s, p_o)$$

IDP in plane wave domain



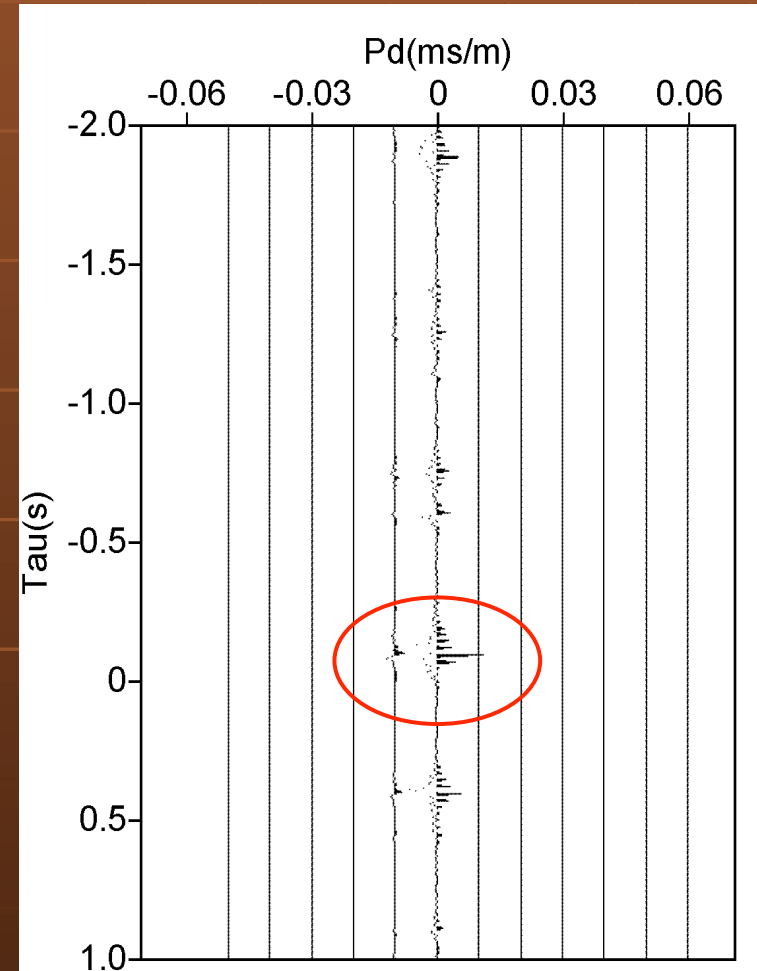
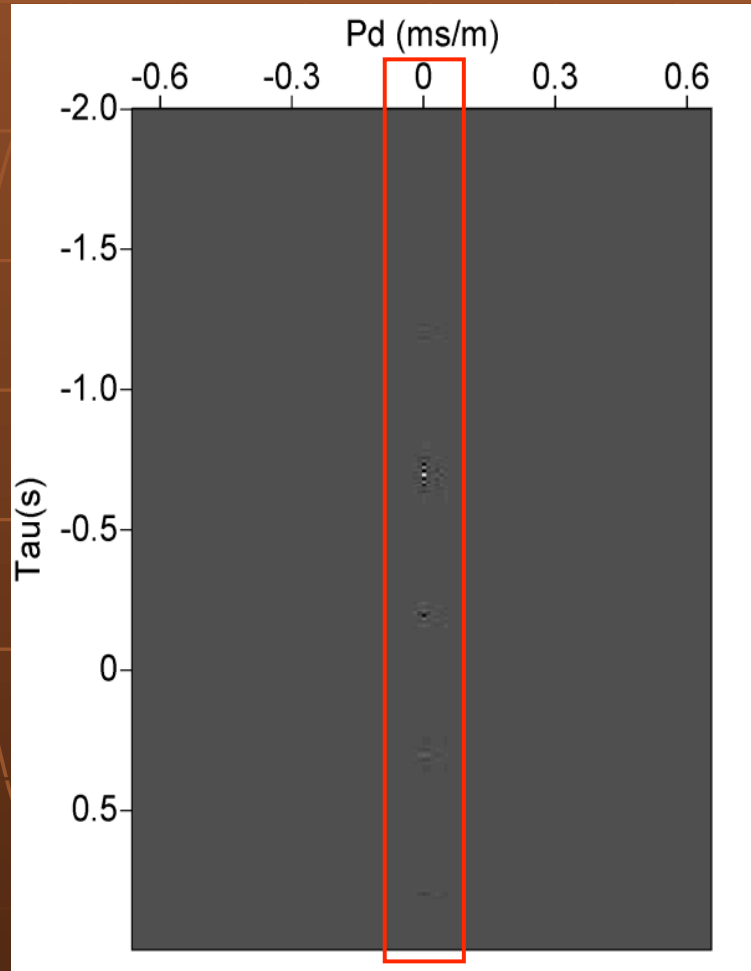
Workflow of *plane wave* domain inverse data processing

IDP in plane wave domain



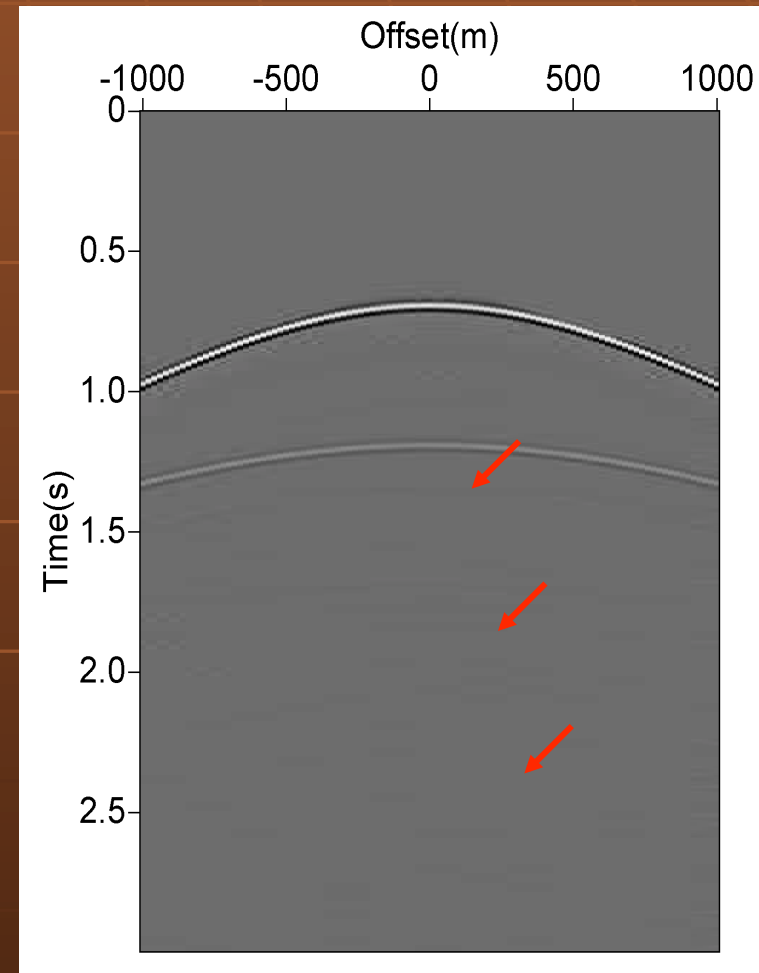
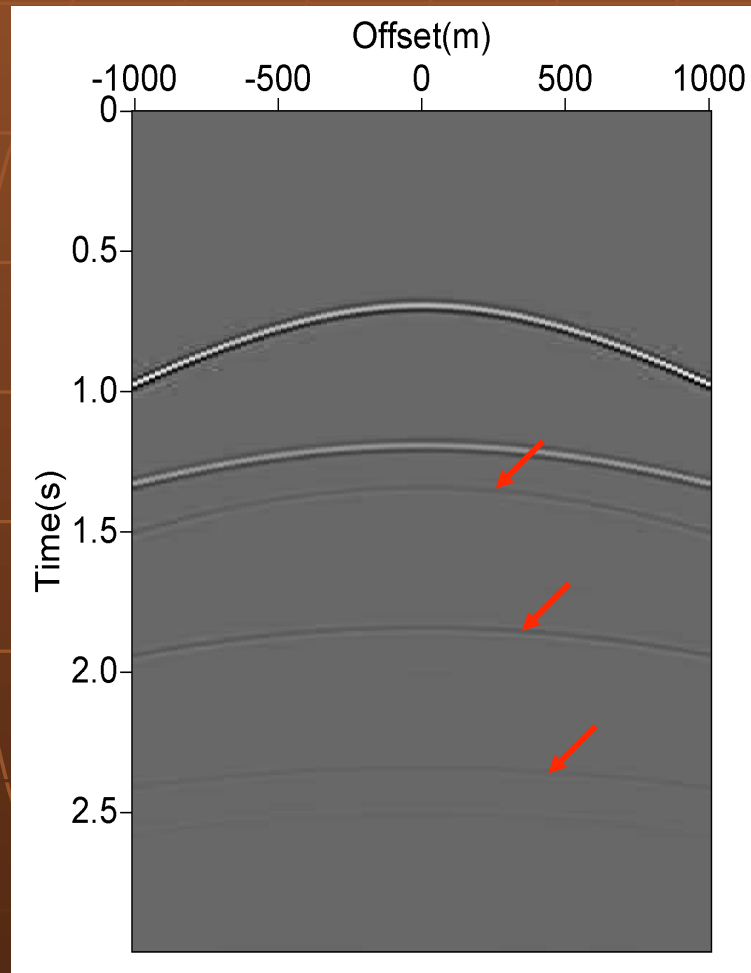
1D shot gather and its 1D and 2D Tau-p transform

IDP in plane wave domain



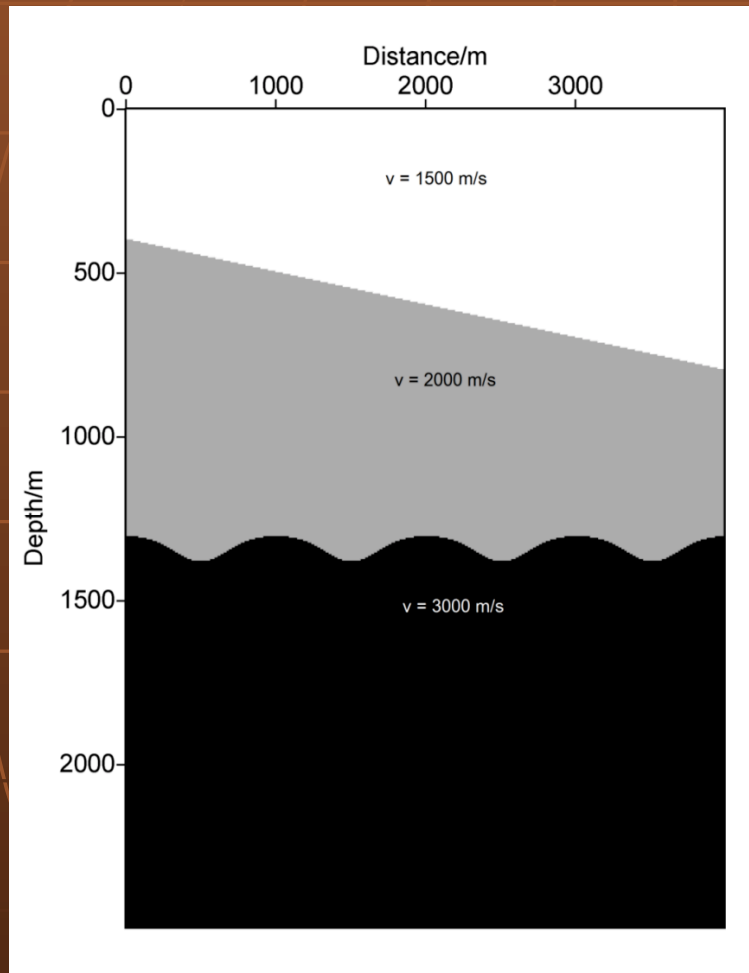
Data in the inverse data space in the plane wave domain

IDP in plane wave domain

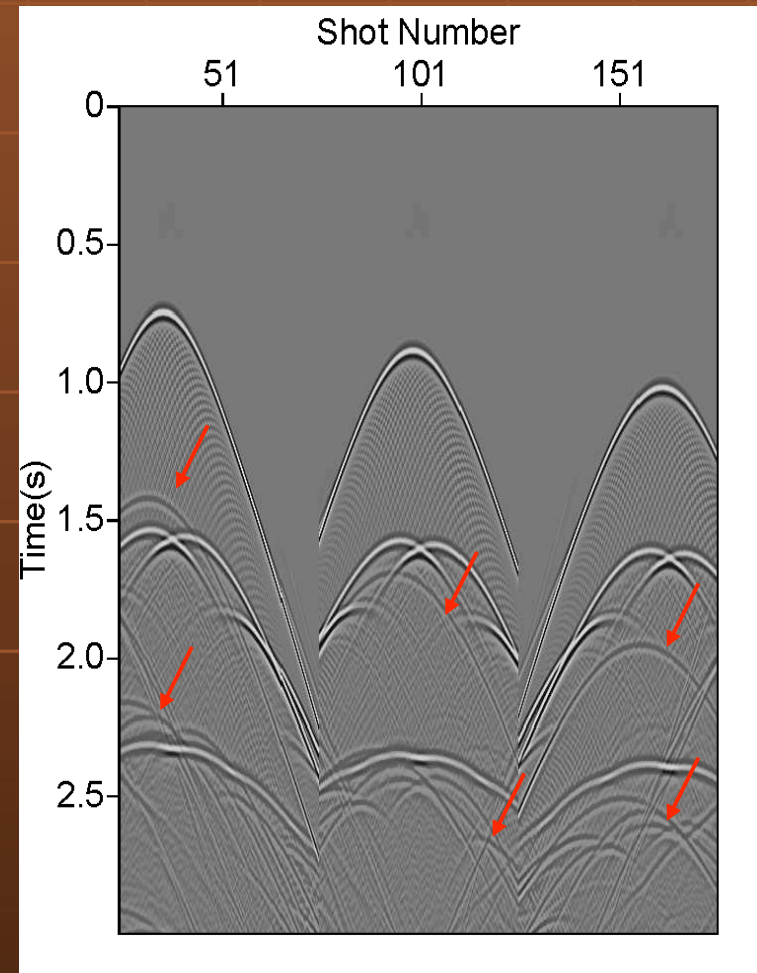


Comparison of data before and after multiple elimination using IDP

IDP in plane wave domain

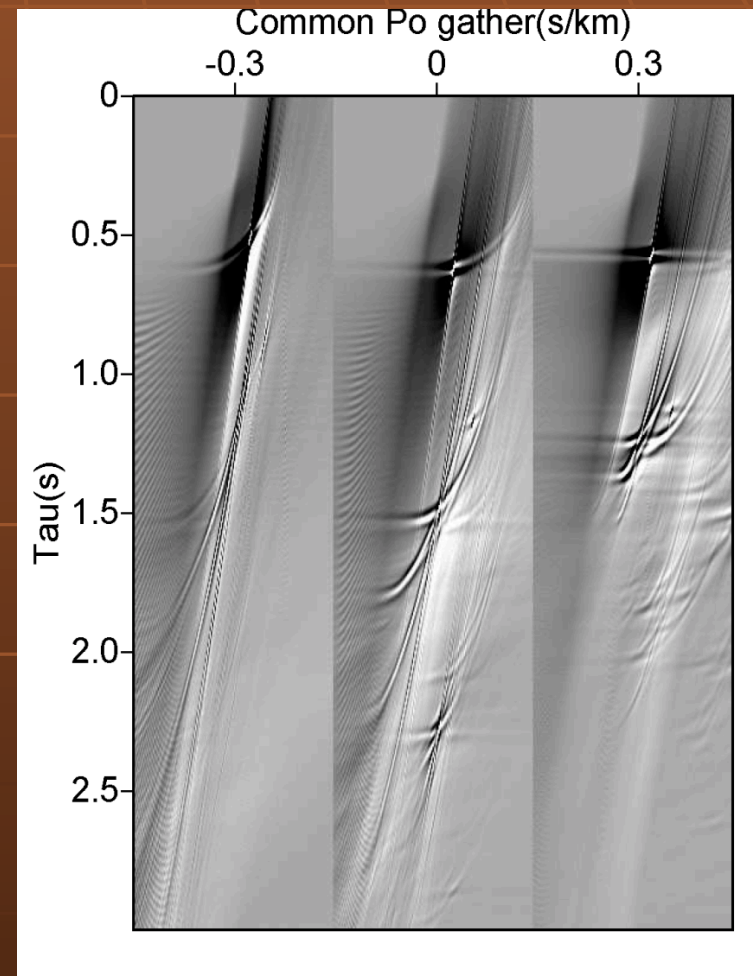
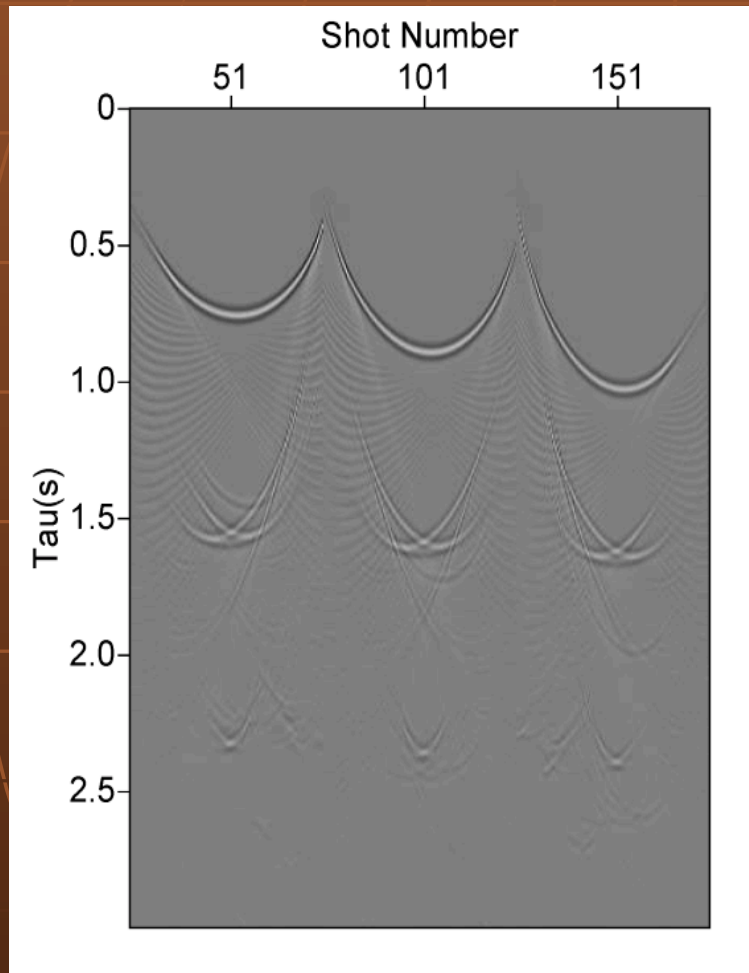


Velocity model



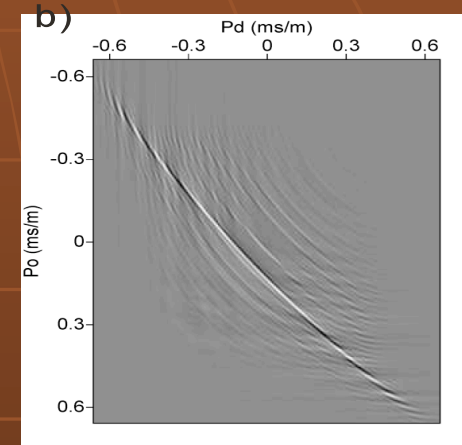
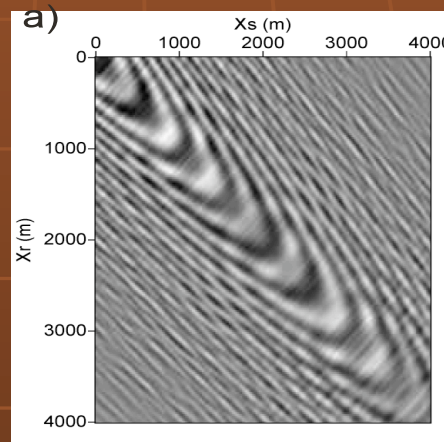
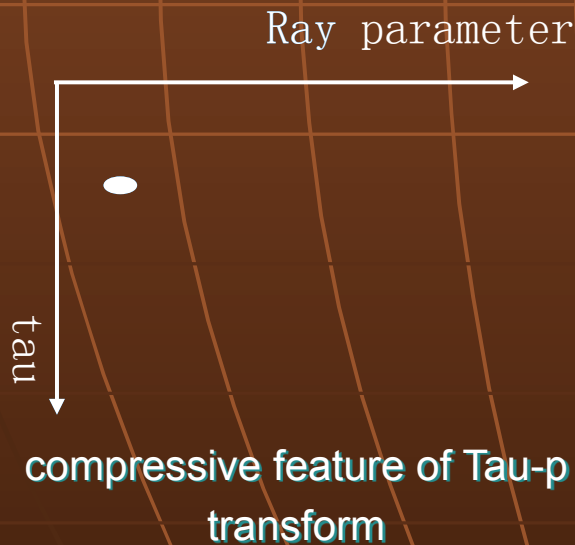
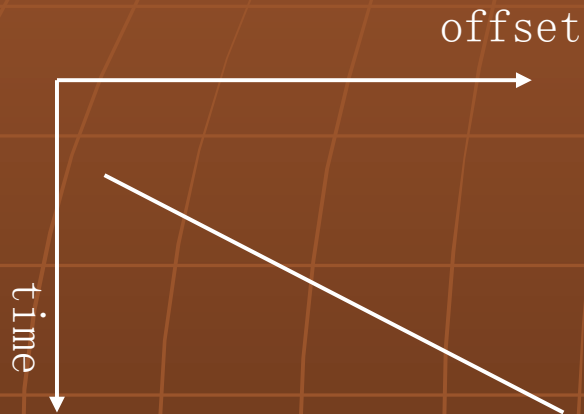
Synthetic shot gathers

IDP in plane wave domain

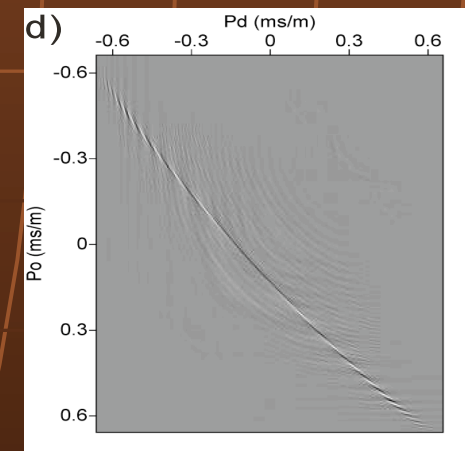
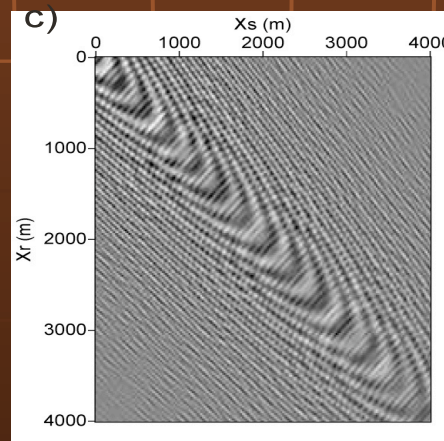


Data in the 1D and 2D Tau-p domain

IDP in plane wave domain

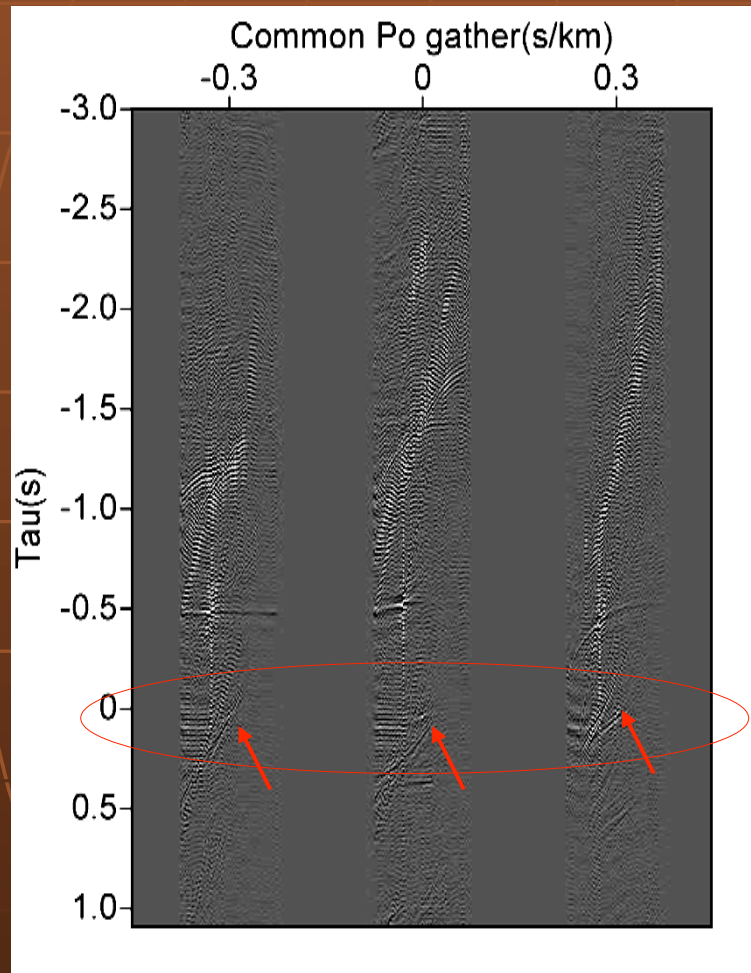


15Hz data matrix



25Hz data matrix

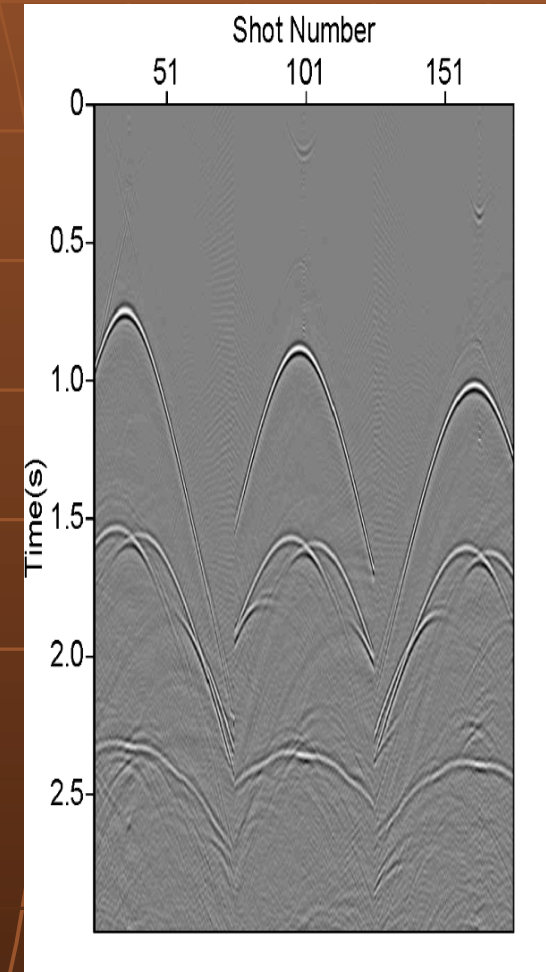
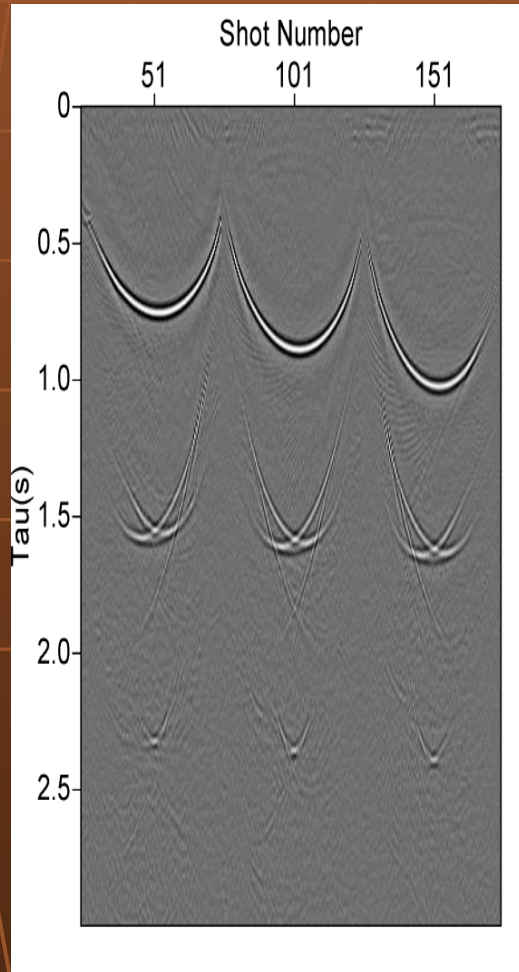
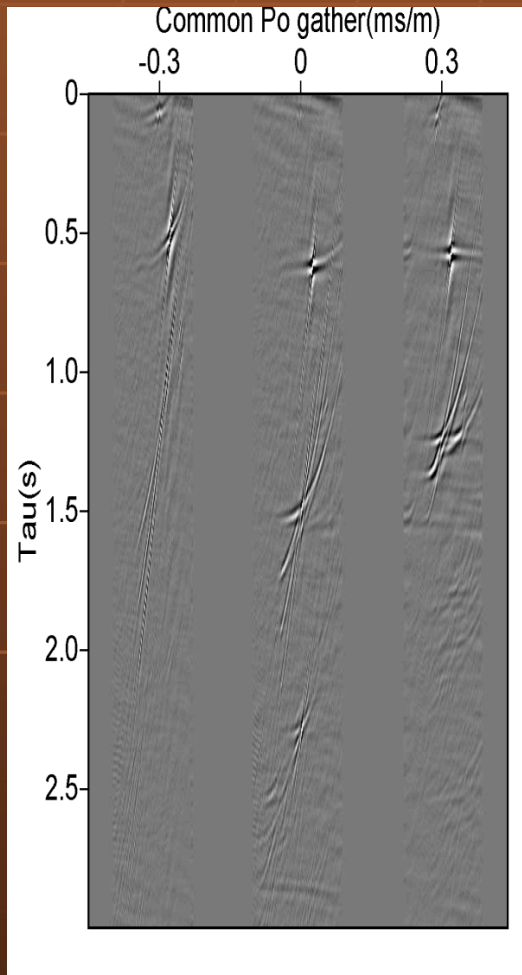
IDP in plane wave domain



Data in the inversed
plane wave domain.

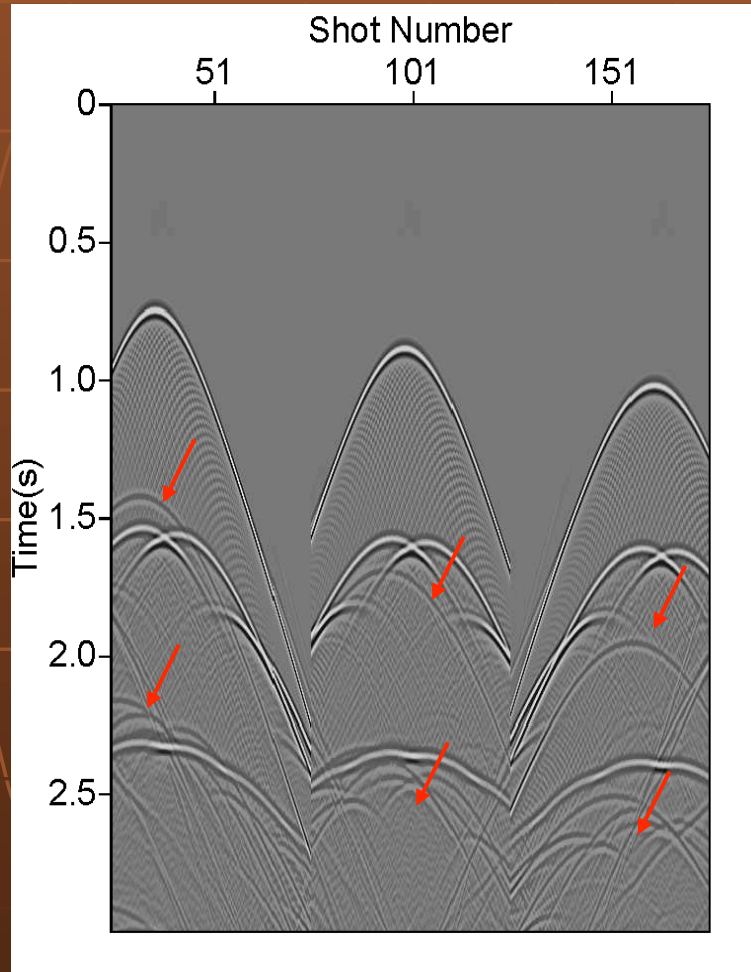
Arrows points at the
focused point, namely,
multiples.

IDP in plane wave domain

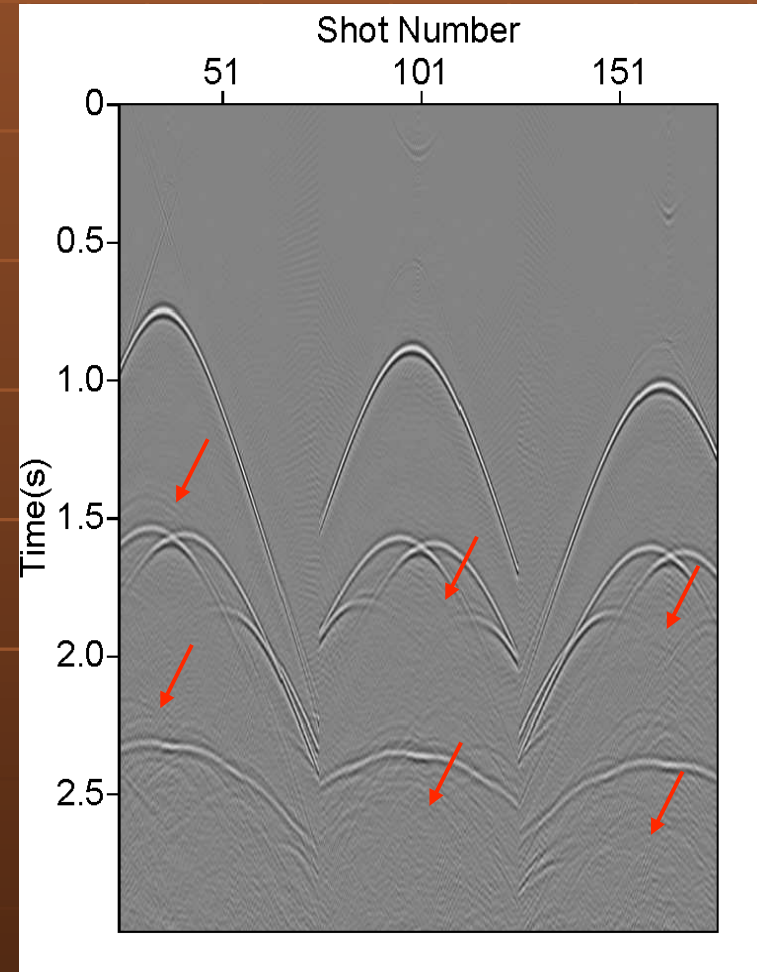


Demul results in Tau-p and x-t domain

IDP in plane wave domain

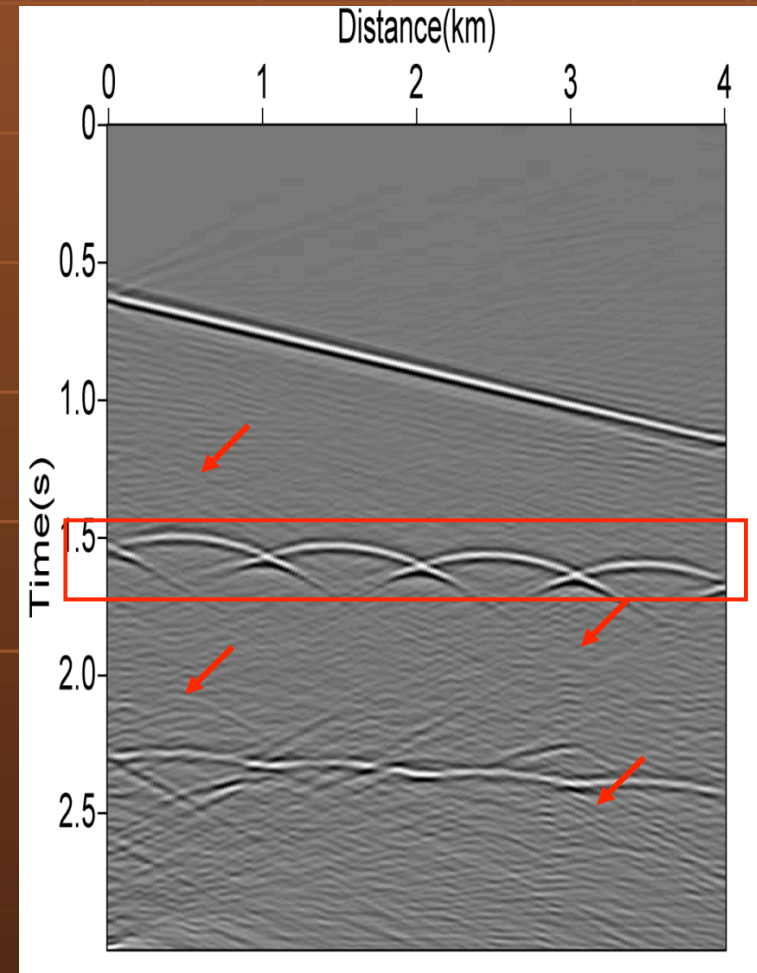
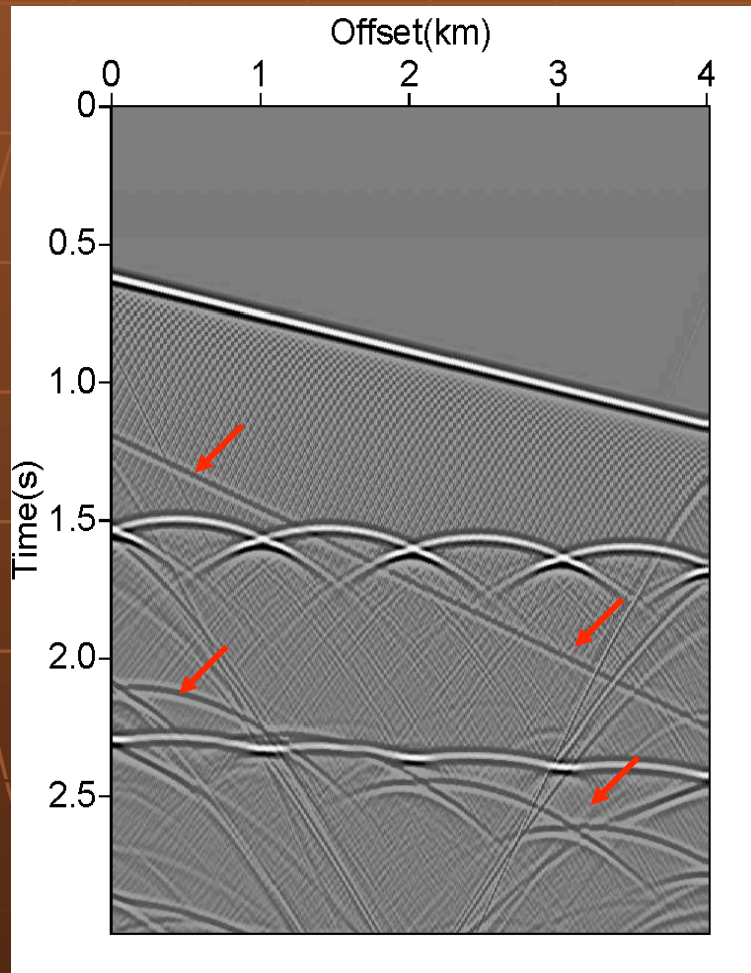


Original shot gathers



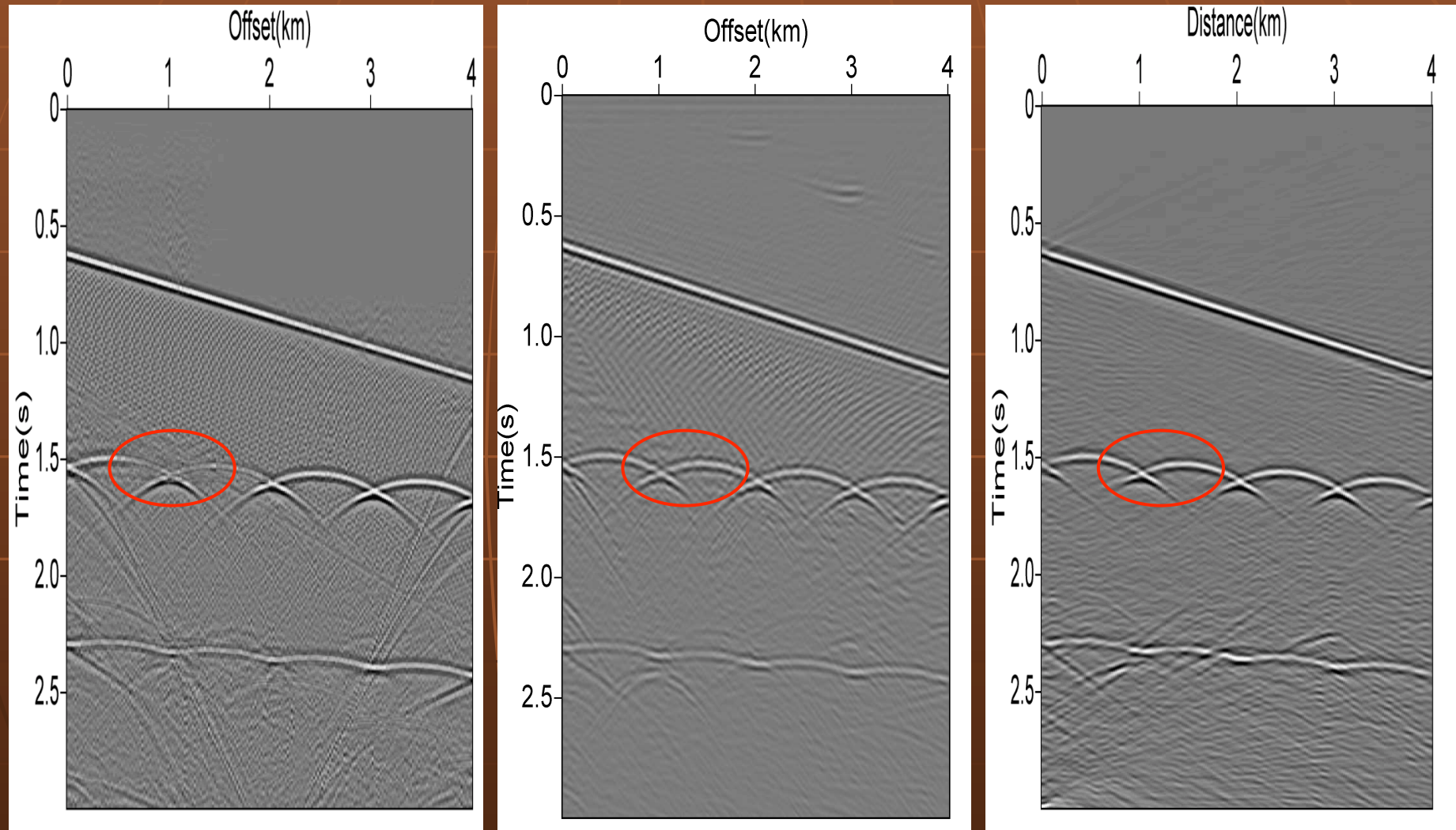
Demul Results

IDP in plane wave domain



Common offset sections comparison before and after demul

IDP in plane wave domain

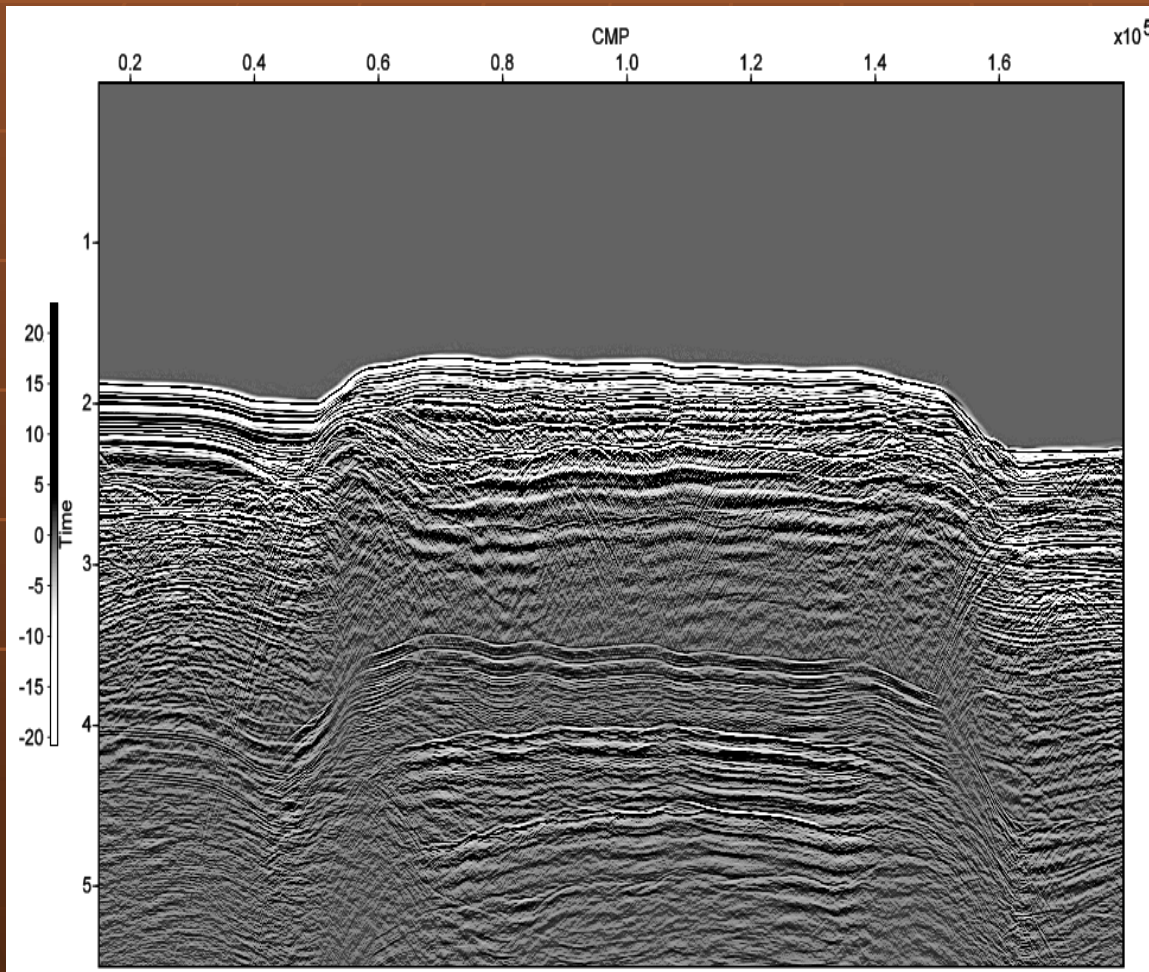


Comparison of three different methods: prediction-subtraction(left), x-t domain inverse₆ data processing (middle) and plane wave domain inverse data processing(right)

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Field data example



Data stack section

1001 shot gathers

180 channels per shot

1501 samples per trace

Shot interval: 26.67m

receiver interval:

26.67m

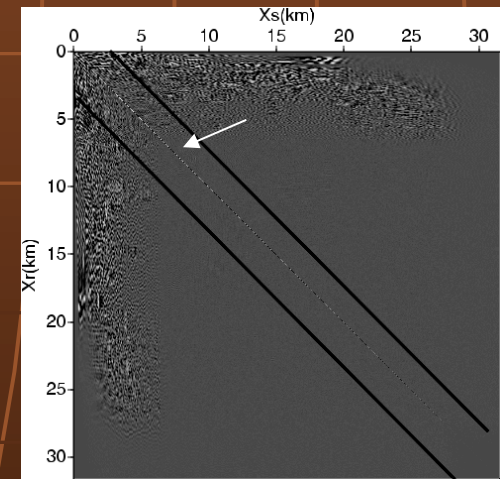
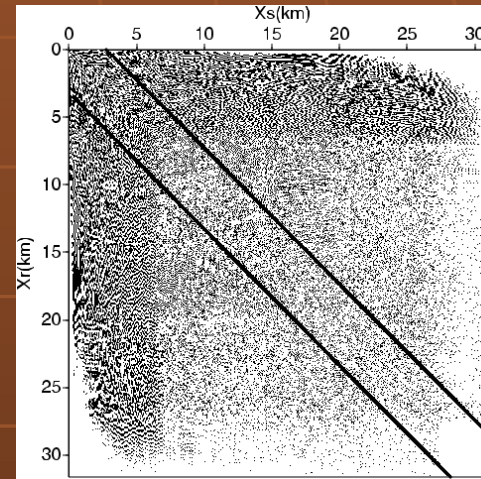
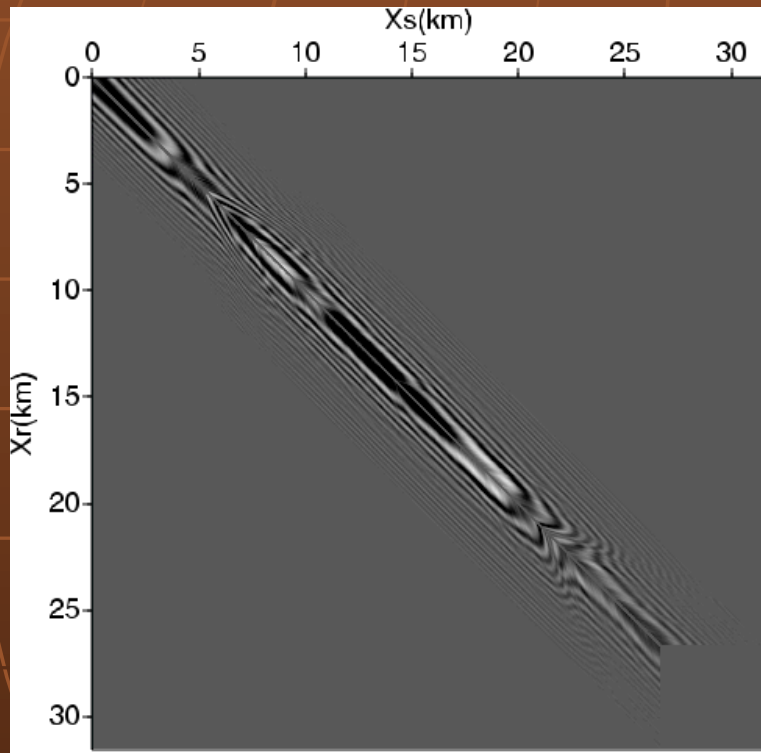
Offset: -100 ~

4874.36m

Sample rate: 4ms

Field data example

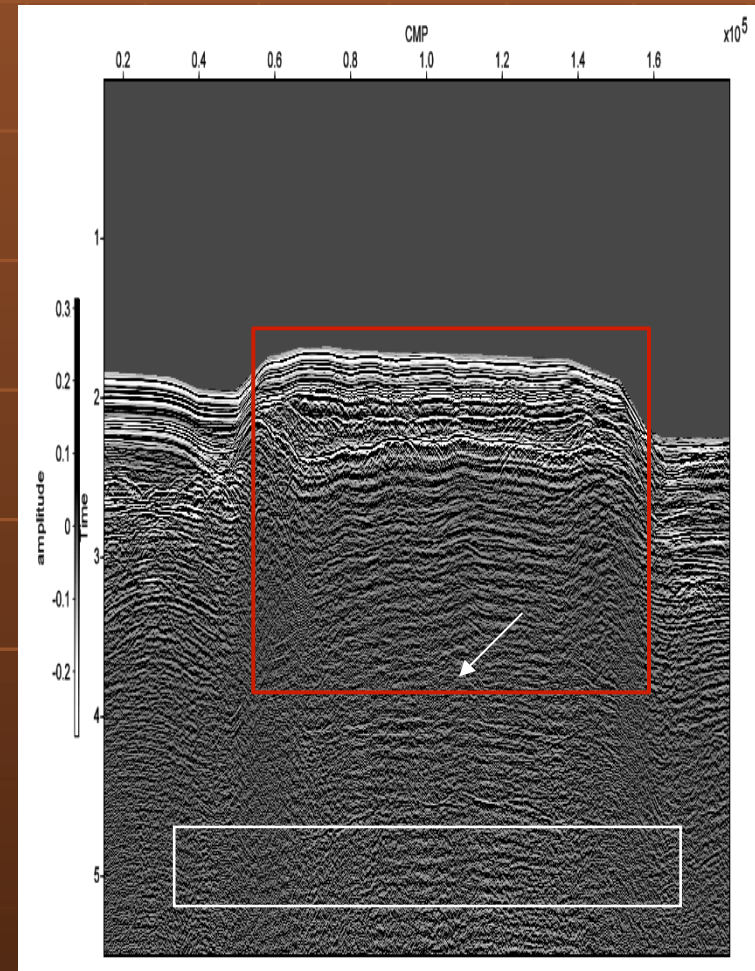
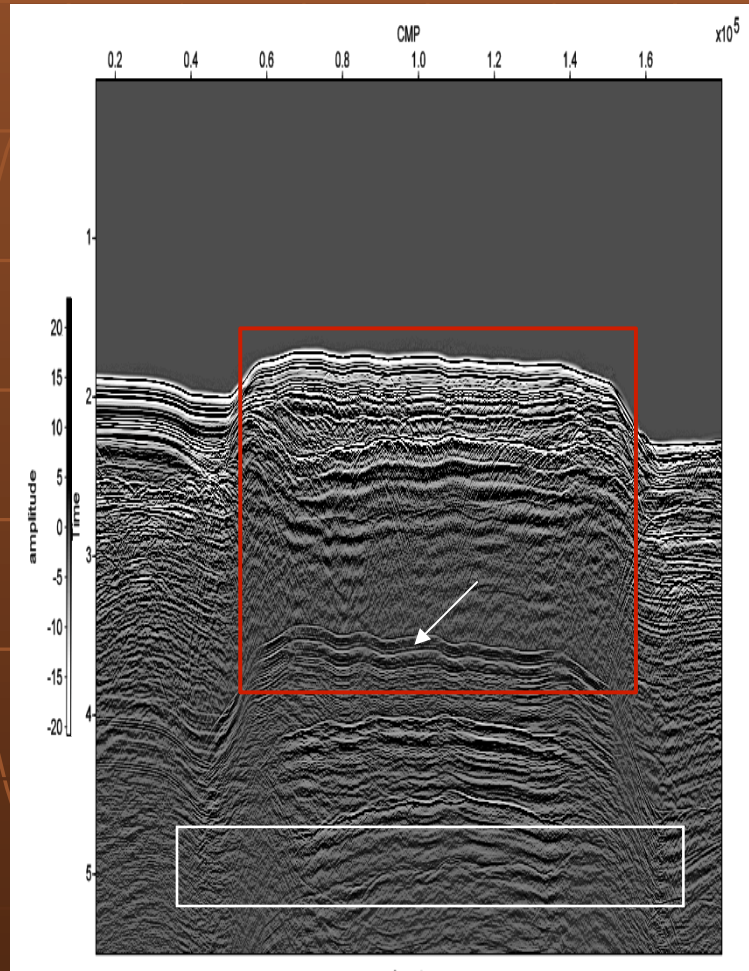
x-t inverse



Data Matrix before and after Inversion

Field data example

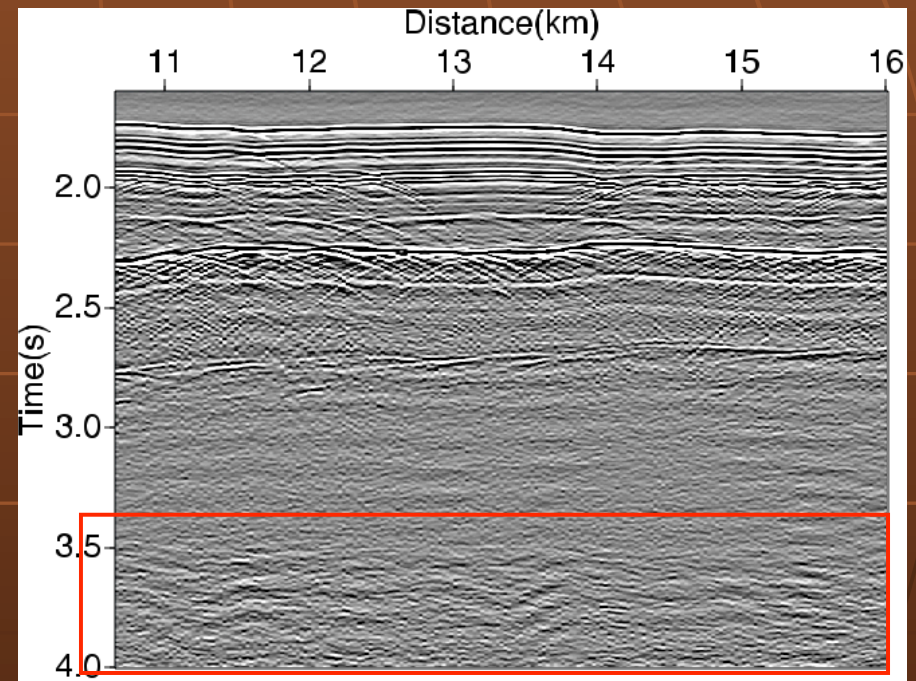
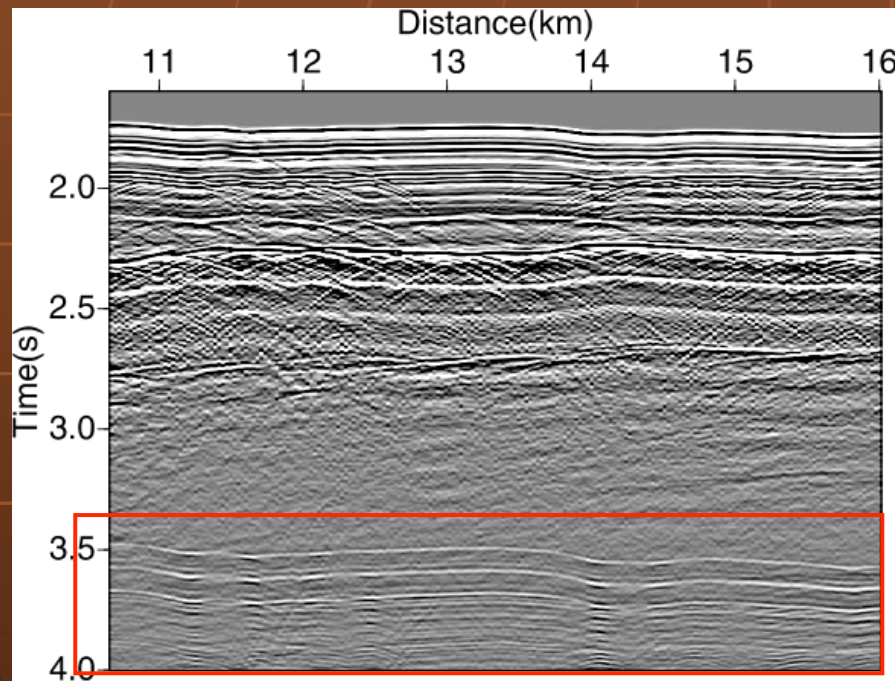
x-t inverse



Data stack section before and after IDP

Field data example

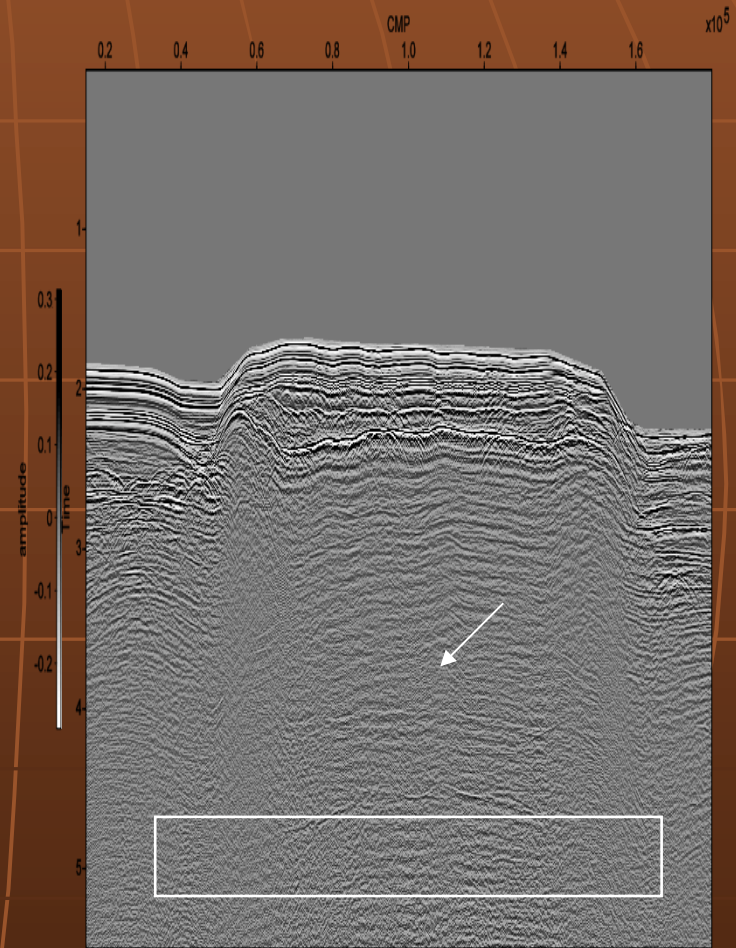
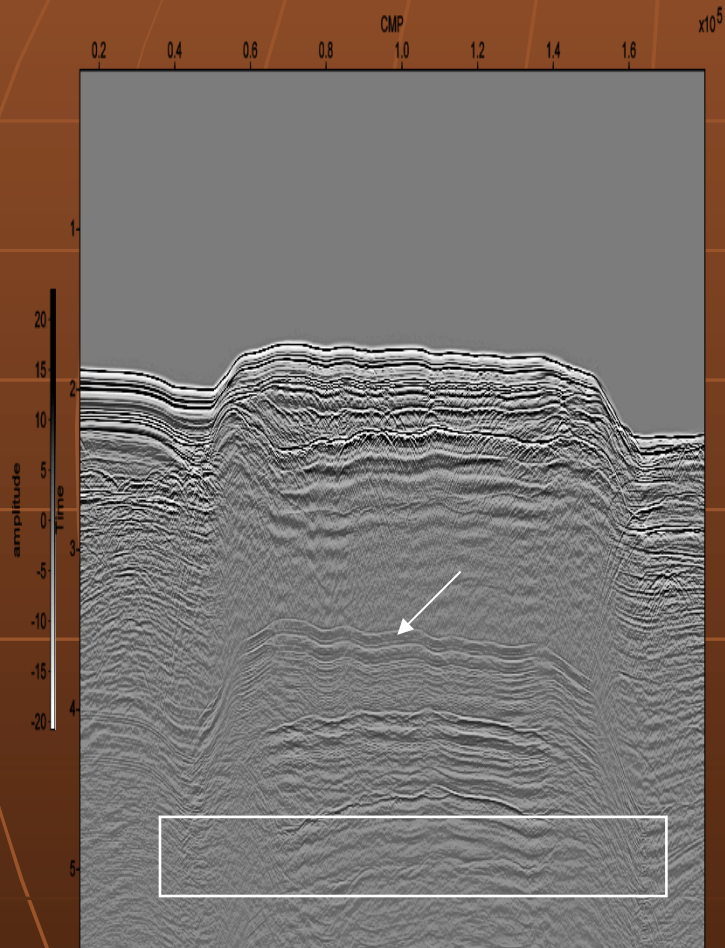
x-t inverse



Zoomed part of stack section

Field data example

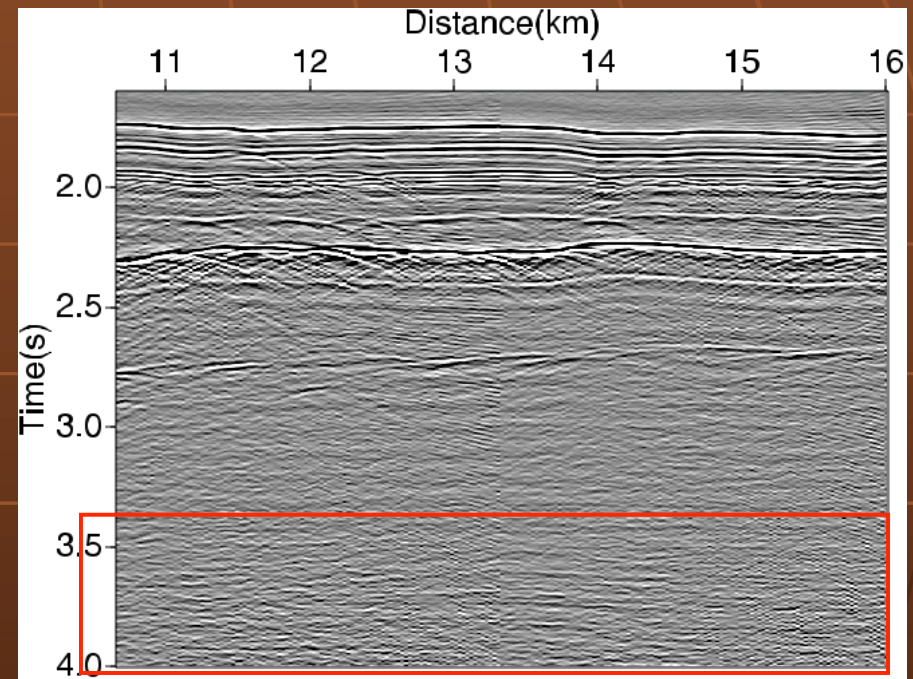
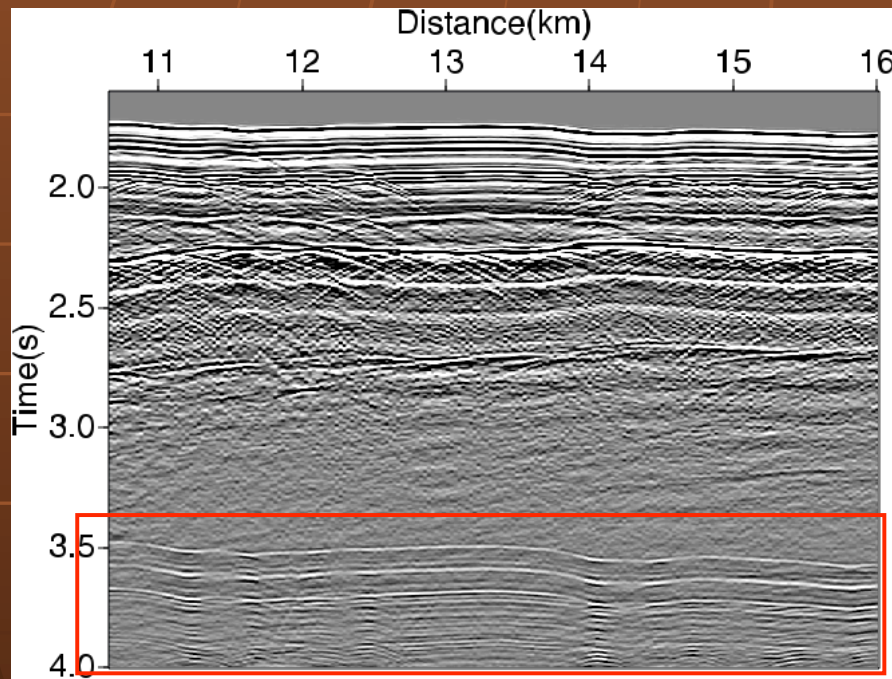
plane wave inverse



Stack section

Field data example

plane wave inverse



Zoomed part of stack section

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wave domain
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Discussion

Inverse Data Processing

- Noise and artifacts are introduced. **More advanced inversion method** is needed.
- The method can be extended to 3D using super-matrices. **Accurate inversion method** will have to be developed to deal with coarse cross-line sampling problem.
- The method can be extended into **internal multiple attenuation** using a **re-datuming** technique.

Conclusion



1. Prediction-subtraction method can predict multiples well; subtraction may damage primary energy.
2. IDP is completely data driven. Fully covered data is needed to carry out the process.
3. IDP is an improvement over a prediction-subtraction method. The inverse data processing can separate multiples and primaries in a very natural way. A simple muting will eliminate all multiples.
4. Tau-p transform technique can compress seismic data efficiently, helping to store more data into memory and making the computation more efficient.
5. 3D extension is applicable.



Thanks!