Seismic Wave Propagation in Fractured Media

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Outline

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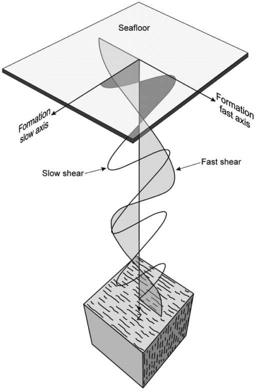
Motivation

- Fractures are a **common feature in the subsurface**,
- Observed in many scales, from faults to micro-cracks,
- Parallel micro-cracks introduce **seismic anisotropy** (Schoenberg & Douma, 1988),
- Characterization of the orientation and density of fractures has important practical applications (Sayers, 2007).

- Two approaches to incorporate the effects of fractures in wave propagation:
  1. Using equivalent media theories,
  2. Simulating the fractures in a numerical scheme.



(Barton, 2007)

Goals

1. Develop a numerical approach to incorporate fractures in wave-propagation simulations,
2. Validate the Equivalent Media Theories,
3. Investigate numerically the sensitivity of the data to the fracture parameters.

Equivalent Media Theories

- Equivalent Media Theories predict the **effective elastic properties** of fractured media given some fracture parameters.
- Common assumptions:
  - Idealized crack shape,
  - small aspect ratio and crack density compared to wavelength,
  - Cracks are isolated with respect to fluid flow.
- Examples of Effective Media Theories:
  - Kuster-Toksöz,
  - Differential Effective Medium,
  - Hudson,
  - Eshelby-Cheng.

(Mavko et al., 1998; Saenger et al., 2004, and references therein)

Numerical Approaches

- Approaches that have been proposed in the literature:
  - Use locally an effective medium (Vlastos et al., 2003),
  - Incorporate locally a low velocity and low density inclusion into a finite difference scheme (Saenger & Shapiro, 2002; Saenger et al., 2004), and
  - Explicitly use a displacement discontinuity condition using the linear-slip model (Zhang, 2005; Zhang & Gao, 2009).
- The advantage: they require few assumptions and therefore they have a broad applicability and are useful to validate the equivalent medium theories.
- Approaches based on the linear-slip model require the least number of assumptions.

The Linear Slip Model

Linear-Slip Model (LSM): Prescribes a linear relation between the traction vector and jump in the displacement:

$$[\boldsymbol{u}] = \boldsymbol{Z}\boldsymbol{\tau}, \tag{1}$$

where $[\boldsymbol{u}]$ is the jump of the displacement, $\boldsymbol{\tau}$ is the traction vector at the fracture and $\boldsymbol{Z}$ is the fracture compliance matrix. For a fracture with up-down symmetry and rotational symmetry about the normal, the fracture compliance matrix is given by (Schoenberg & Douma, 1988; Zhang & Gao, 2009)

$$\boldsymbol{Z} = \begin{pmatrix} Z_T & 0 & 0 \\ 0 & Z_T & 0 \\ 0 & 0 & Z_N \end{pmatrix}, \tag{2}$$

where Z_T and Z_N are the tangential and normal components of the compliance matrix.

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Discontinuous Galerkin Method

- The Discontinuous Galerkin Method (DGM) is a generalization of FEM that allows for the basis functions to be discontinuous at the element interfaces.
- IP-DGM: Interior-penalty formulation
  - SIPG: Symmetric Interior Penalty Galerkin (Darlow, 1980),
  - NIPG: Non-symmetric (Rivière & Wheeler, 2001),
  - IIPG: Incomplete (Dawson et al., 2004).
- Advantages
  - it can accommodate discontinuities in the wave field,
  - it can be energy conservative,
  - it can handle more general meshes, and
  - it is suitable for local time stepping and parallel implementations.

Accuracy and Stability of DGM

- Grid dispersion and stability analyzed in De Basabe et al. (2008) and De Basabe & Sen (2010).
- **Superconvergence** of the grid-dispersion error with respect to the sampling ratio for the symmetric formulation and nodal basis functions,
- The numerical **anisotropy** is negligible for basis of degree 4 or greater,
- **Stability** condition in 2D given by

$$\frac{\alpha \Delta t}{\Delta x} \leq 0.25,$$

where Δx is the smallest spatial increment, Δt is the size of the time step and α is the largest P-wave velocity.

Interior-Penalty Weak Formulation

Find $\boldsymbol{u} \in \boldsymbol{X}^D$ such that for all $\boldsymbol{v} \in \boldsymbol{X}^D$

$$\sum_{E\in\Omega_h} \Big((\rho\partial_{tt}\boldsymbol{u}, \boldsymbol{v})_E + \boldsymbol{B}_E(\boldsymbol{u}, \boldsymbol{v}) \Big) + \sum_{\gamma\in\Gamma_h} \boldsymbol{J}^c_\gamma(\boldsymbol{u}, \boldsymbol{v}; S, R) = \sum_{E\in\Omega_h} (\boldsymbol{f}, \boldsymbol{v})_E$$

where $X^D = \Big\{ \varphi \mid \varphi \in \boldsymbol{H}^1(E)\ \forall\ E \in \Omega_h,\ \boldsymbol{\varphi} = 0 \text{ on } \Gamma_D \Big\}$

$$\begin{aligned}
\boldsymbol{B}_E(\boldsymbol{u}, \boldsymbol{v}) &= \int_E \Big(\lambda\partial_i u_i \partial_j v_j + \mu(\partial_j u_i + \partial_i u_j)\partial_j v_i \Big)\, d\Omega, \\
\boldsymbol{J}^c_\gamma(\boldsymbol{u}, \boldsymbol{v}; S, R) &= -\int_\gamma \{\tau_i(\boldsymbol{u})\}[v_i]\, d\gamma + S\int_\gamma \{\tau_i(\boldsymbol{v})\}[u_i]\, d\gamma \\
&+ R\int_\gamma \{\lambda + 2\mu\}[u_i][v_i]\, d\gamma, \\
\tau_i(\boldsymbol{u}) &= \sigma_{ij}(\boldsymbol{u})n_j = \lambda u_{k,k} n_i + \mu(u_{i,j} + u_{j,i})n_j.
\end{aligned}$$

The parameter R is the penalty, and S is a parameter that takes the values $S = 0$ for IIPG, $S = -1$ for SIPG and $S = 1$ for NIPG.

Proposed Numerical Scheme

Find $\boldsymbol{u} \in \boldsymbol{X}^D$ such that for all $\boldsymbol{v} \in \boldsymbol{X}^D$

$$\sum_{E\in\Omega_h} \Big((\rho\partial_{tt}\boldsymbol{u}, \boldsymbol{v})_E + \boldsymbol{B}_E(\boldsymbol{u}, \boldsymbol{v}) \Big) + \sum_{\gamma\in\Gamma_c} \boldsymbol{J}^c_\gamma(\boldsymbol{u}, \boldsymbol{v}; S, R) + \sum_{\gamma\in\Gamma_f} \boldsymbol{J}^f_\gamma(\boldsymbol{u}, \boldsymbol{v}; S, R) = \sum_{E\in\Omega_h} (\boldsymbol{f}, \boldsymbol{v})_E$$

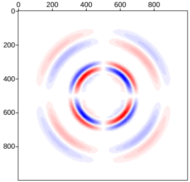
where $\Gamma_c \subset \Gamma_h$ is the subset of all faces where the displacement field is continuous, $\Gamma_f \subset \Gamma_h$ is the subset of faces that represent fractures, and

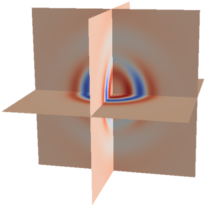
$$\begin{aligned} \boldsymbol{J}^f_\gamma(\boldsymbol{u}, \boldsymbol{v}) &= -\int_\gamma \{\tau_i(\boldsymbol{u})\}[v_i]\, d\gamma \\ &+ R\int_\gamma \{\lambda + 2\mu\}([\boldsymbol{u}] - \boldsymbol{Z}\{\boldsymbol{\tau}_\gamma(\boldsymbol{u})\}) \cdot ([\boldsymbol{v}] - \boldsymbol{Z}\{\boldsymbol{\tau}_\gamma(\boldsymbol{v})\})\, d\gamma. \end{aligned}$$

The linear slip condition is weakly imposed through the penalty term.

SWP

- The Seismic Wave Propagation software (SWP) is a computer code written in C++ designed to simulate acoustic or elastic wave propagation in 2D and 3D.
- The main characteristic of this software is that it encapsulates many methods for discretizations in space and time of the acoustic or elastic wave equation and, therefore, it is useful to compare the accuracy and performance of the methods.
- We have added the Linear Slip Model to this software using IP-DGM.





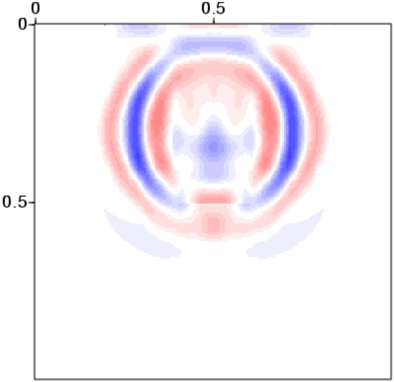
Methods available in SWP

Method	Version	Time-stepping Methods
2D SEM	Acoustic	FDM, RK-4, LWM-4
	Elastic	FDM, RK-4, LWM-4
	Acoustic-Elastic	FDM, RK-4
2D IP-DGM	Acoustic	FDM, RK-4, LWM-4
	Elastic	FDM, RK-4, LWM-4
	Acoustic-Elastic	FDM, RK-4
2D SG-FDM	Elastic	FDM
3D SEM	Acoustic	FDM, LWM-4
	Elastic	FDM, LWM-4
3D IP-DGM	Acoustic	FDM, LWM-4
	Elastic	FDM, LWM-4

- SG-FDM is the 4th order staggered grid FDM, RK-4 is the 4th order Runge-Kutta method and LWM-4 is the 4th order LWM.
- The polynomial degree of the basis functions used in SEM and IP-DGM can be between 1 and 10.

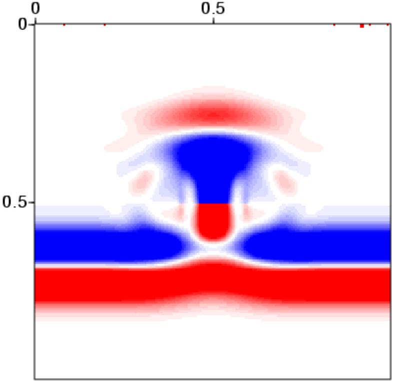
Preliminary Results

- Domain: 1 Km^3,
- Periodic boundary conditions in x and y, free surface on z,
- Point source at (0.5, 0.5, 0.3),
- Horizontal fracture centered at (0.5, 0.5, 0.5),
- $V_P = 3.231$ Km/s, $V_S = 1.96$ Km/s, $\rho = 2.44$ gr/cm^3



Preliminary Results

- Domain: 1 Km^3,
- Periodic boundary conditions in x and y, free surface on z,
- Horizontal plane wave,
- Horizontal fracture centered at (0.5, 0.5, 0.5),
- $V_P = 3.231$ Km/s, $V_S = 1.96$ Km/s, $\rho = 2.44$ gr/cm^3



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Ongoing and Future Work

- Ongoing Work:
  - We have developed a 3D elastic wave propagation code that can incorporate fractures.
  - We are currently testing the code, comparing with analytic results for simple cases (homogeneous media with one linear-slip discontinuity).
- Future Work:
  - Perform numerical experiments with more complicated models: Parallel-horizontal fractures, intersecting fractures, etc.,
  - Systematically compare the Equivalent Media Theories using low and increasingly high fracture densities,
  - Systematically evaluate the sensitivity of the synthetic seismograms to the fracture parameters.

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